

**MINISTRY OF EDUCATION AND TRAINING
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**TEACHING ANALYTICS AT HIGH SCHOOLS IN
THE DIRECTION OF IMPROVING PROBLEM SOLVING
COMPETENCE THROUGH EQUIPPING WITH SOME TACTICS
OF COGNITIVE ACTIVITIES FOR STUDENTS**

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- National Library of Vietnam
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LIST OF THE AUTHOR'S PUBLICIZED WORKS RELATED TO THESIS TOPIC

1 Books:

1. Think Thi Bach Tuyet (2012), "*Apply the variance of function to find out root of equation*", *Selection of special subjects of Mathematics and youth*, volume 6, Vietnam Education Publishing House, p. 34-36.
2. Think Thi Bach Tuyet (2014), "*A small technique to solve the equation $A > 0$* ", *Selected special subjects for preparation of graduation exam from high school and entrance exam in Universities and Colleges*, Volume, Algebra, Trigonometry, Analytics, Vietnam Education Publishing House, p. 129-132.

2 Articles:

1. Think Thi Bach Tuyet (2013), "*Use tactics in teaching solution of mathematics exercises at high schools*", *Education Magazines*, special print in August, p. 86-88.
2. Think Thi Bach Tuyet (2014), "*Use tactics in teaching some concepts of Analytical Mathematics at High schools*", *Educational Science Magazine*, special print, p. 4-6.
3. Think Thi Bach Tuyet (2014), "*Use tactic of symbolization in teaching the concept of Analytics at High Schools*", Summary record of national scientific conference, Research of mathematical science in the direction of developing learners' competence, period 2014-2020, Publishing House of Hanoi National University of Education, p. 141-146.
4. Think Thi Bach Tuyet (2015), "*Apply tactic of function graph to teach solution of mathematical exercises at high schools*", Summary record of scientific conference, Developing occupational competence of mathematics teachers at high schools in Vietnam, Publishing House of Hanoi National University of Education, p. 187-192.
5. Think Thi Bach Tuyet (2015), "*Establish tactic of cognitive activities for students in teaching Mathematics at high schools*", *Scientific Magazine*, Hanoi National University of Education, p. 198-204.

PREAMBLE

1. Rationale

1.1 Establishment and development of problem solving capacity for students are important targets of mathematics

Problem solving has important meaning in teaching mathematics and has been applied in curricula in many countries worldwide. Researching the relationship between the contents of Mathematics at high schools in Vietnam and common capacities which should be established and developed for students, Tran Kiel determines that problem solving competence is one of six specific capacities which should be established and developed for students through Mathematics. Therefore, the improvement of problem solving competence has currently been one of significant duties in teaching mathematics at high school in Vietnam.

1.2 Analytical content is potential to improve the problem solving competence

The analytical content contains many contexts which arise out of problems and is an aspect which could be exploited to improve the problem solving competence.

1.3 The tactics of cognitive activities play the significant role against the students regarding apprehension of mathematical knowledge as well as solution of mathematical problems

Polya affirmed that teaching tactics (referred to as tactics of cognitive activities) is to develop the problem solving competence for students. In practical mathematics teaching, featured and skilful methods of surveying and changing objects to find out reasonable and optimal measures shall help the students perceive the beauty of mathematics, establish aesthetic feeling and inspire the passion for and interest in mathematics. Such methods play the role as means or tools which help the students occupy the whole knowledge of mathematics and solve successfully mathematical problems. And such methods are considered as tactics of cognitive activities.

Equipping with tactics of cognitive activities for the students in teaching analytics is very necessary and considered as one of ways which contribute to establish and develop the problem solving competence.

We have selected the research topic: ***“Teaching analytics at high schools in the direction of improving problem solving competence through equipping with some tactics of cognitive activities for students”*** therefrom.

2. Overview

2.1 Some researches of tactics and tactics of cognitive activities

Some researches of tactics of cognitive activities have shown that when the tactics of cognitive activities are equipped with, the grasp of problems is more efficient; the tactics of cognitive activities are made use of in the duration of problem solving; the tactics of cognitive activities is the efficient tool to bring concepts, knowledge and skills in problem solving; the students are required not only to “learn” the tactics but also able to select which tactics is the most suitable in

each period of problem solving. The research of equipping with tactics of cognitive activities in order to improve the problem solving competence is necessary.

2.2 Some researches on improvement of problem solving competence and teaching analytics at high schools

It has shown that researches of teaching analytics in the direction of accessing competence and analyzing the analytical contents at high school are unavailable.

Thereby, the teaching in the direction of establishing and developing competence has currently been the trend of education in Vietnam. There are many really meaningful researches of teaching mathematics in general and algebra and geometry in particular in the direction of improving the problem solving competence at high schools. Analytics is a difficult and important subject which has many applications and researches at high schools; however, researches of teaching analytics in the direction of developing problem solving competence are unavailable. The tactics of cognitive activities is used in problem solving. The research of teaching analytics in the direction of accessing the problem solving competence through equipping with some tactics of cognitive activities has still been leaving open and not mentioned to in any work; therefore, the thesis will research this issue.

3. Purpose of research

Research and seek some measures of equipping with some tactics of cognitive activities for the students in order to improve the problem solving competence and contribute to increase the efficiency of teaching analytics at high schools.

4. Object, subject and scope of research

3.1 Object: Activities of teaching Analytics at high schools.

3.2 Subject of research: Some tactics of cognitive activities in teaching mathematics to improve the problem solving competence for students at high schools.

3.3 Scope of research: Analytical content included in curricula and textbooks at high schools.

5. Scientific assumption

The determination of some tactics of cognitive activities in teaching Analytics and application of reasonable measures to equip with such tactics for the students shall improve the problem solving competence for students and contribute to increase the efficiency of teaching Analytics.

6. Duties of research

The thesis will research the following issues:

- Clarify the problem solving in mathematics; clarify the concept of problem solving competence; Components of problem solving competence; Relationship between the problem solving activity and problem solving competence.

- Summarize some tactics-related researches; Recommend concept of tactics of mathematical cognitive activities; Recommend some specific tactics of mathematical cognitive activities which should be equipped with for the students.

- Research contents and curricula of mathematics in general and analytics in particular at high schools.

- Research the actual status of teaching analytics in the direction of equipping with some tactics of cognitive activities for the students at high schools.

- Recommend pedagogic measures of teaching analytics in the direction of improving problem solving competence for the students through equipping with some tactics of cognitive activities.

- Pedagogically practice to initially check feasibility and efficiency of pedagogic measures recommended by the thesis.

7. Methods of research

Method of theoretical research; Method of survey and observation; Method of pedagogic practice; Method of mathematical statistics in educational science; Professional method.

8. New contributions of the thesis

8.1. Theory

- Clarify issues of problem solving competence and components of problem solving competence.

- Contribute to clarify the concept of mathematical tactics of cognitive activities, some specific tactics of cognitive activities in analytics. Clarify idea of equipping with tactics of cognitive activities and indicative symbols of case using the tactics of cognitive activities.

- Clarify the characteristics of analytical contents at high schools, opportunities of establishing and developing the problem solving competence through teaching analytics, relationship between equipping with tactics of cognitive activities and problem solving competence in teaching analytics.

- Recommend some pedagogic measures to clarify the way of teaching analytics in the direction of improving the problem solving competence through equipping with some tactics of cognitive activities.

8.2. Reality

- Show some restrictions in teaching analytics resulted from the teachers' omission of equipping with some tactics of cognitive activities.

- Offer some specific pedagogic instructions for equipping with some tactics of cognitive activities in teaching analytics. Provide references for teachers, contribute to increase the efficiency of teaching mathematics at high schools.

- Contribute to renovate the method of teaching mathematics, prove the feasibility of teaching analytics in the direction of improving problem solving competence through equipping with some tactics of cognitive activities.

9. Contents

- Concept of tactics of cognitive activities, meaning of tactics of cognitive activities, role of tactics of cognitive activities, identification of tactics of cognitive activities.

- Equip with mathematical tactics of cognitive activities which play important role in teaching analytics in high schools.

- Process of equipping with tactics of cognitive activities has paid reasonable attention to the increase of efficiency of teaching analytics and contributed to improve components of problem solving competence in specific cases such as concept, learning theorem, rules and methods, and applying analytical knowledge.

- Pedagogic measures of teaching analytics in the direction of improving problem solving competence through equipping with some tactics of cognitive activities are feasible and effective.

Chapter 1. THEORY AND PRACTICE

1.1 Problem solving competence

1.1.1 Teaching problem solving

1.1.1.1. Problem in teaching mathematics

Problem in teaching mathematics at high schools is a requirement. The students must acknowledge the necessity, desire and be active to find out the solution.

Problem in teaching mathematics at high schools is the one which the students does not know the solution but have sufficient knowledge and necessary skills to solve.

1.1.1.2 Problem-arousing case

The problem-arousing case is an available problem which the students desire to solve and believe that they could solve.

1.1.1.3 Teaching problem solving

Teach the problem solving in order to develop the students' competence of cognition, particularly the thinking and problem solving competence. Teaching the problem solving is aimed at establishing the problem solving competence which plays the significant role so that the people could adapt with to the development of the society in the future.

1.1.2 Process of problem solving

The process of problem solving includes four steps as follows: Step 1. Survey and acknowledge the problem; Step 2. Seek measures; Step 3. Execute measures; Step 4. Research deeply measures.

1.1.3 Problem solving competence

1.1.3.1 Competence

The students' mathematics competence is their ability of applying knowledge, skills, experience and other personal qualifications such as will, faith... to satisfy with complicated requirements and execute successfully their duties in mathematics activities.

1.1.3.2 Mathematics capacity

- The mathematics competence includes psychological characteristics regarding the students' intelligence activities, helping them grasp thoroughly and apply relatively quickly, easily, deeply knowledge and skills in mathematics.

- The mathematics competence is established, developed and shown through (and attached to) the students' activities in order to solve duties in learning mathematics: establish and apply concepts, prove and apply theorems; solve mathematics problems...

1.1.3.3 Competence of problem solving

The students' problem solving competence is their ability of applying knowledge, skills, experience and other personal qualifications to realize the problem solving activity when they must face with mathematics problems where the way of finding out any solution is not clear and immediate.

1.1.3.4 Components of problem solving competence

The problem solving competence includes 4 components as follows: Competence of understanding problem; Competence of finding out measures; Competence of realizing measures; Competence of researching deeply measures.

1.1.3.5 Relationship between problem solving activity and problem solving competence

The problem solving competence is shown through results of problem solving activities which expose the problem solving competence. Therefore, the establishment and development of the problem solving competence require the students to realize the the problem solving activities.

1.2 Tactics of cognitive activities

1.2.1 Viewpoint

Activity is the process of mutual conversion between the subject and the object. The activity is always aimed at affecting and changing or receiving anything. The objective activity is to form any product in relation to the satisfaction with demands of the people and the society.

1.2.2 Cognitive activities

The mathematics cognitive activity is the process leading to the apprehension of mathematics knowledge, grasping meaning of such knowledge: Determine cause-effect relationship and other relationships of researched mathematics subjects (concepts, relations; mathematics rules...); then apply the mathematics knowledge to solve any practical problem.

1.2.3 Methodological knowledge under the viewpoint of activities

In consideration with the viewpoint of activities, the methodological knowledge in teaching mathematics are the ones of methods of realizing the mathematics cognitive activities. They are specially knowledge of realizing activities of apprehending mathematics knowledge, understanding mathematics knowledge and applying mathematics knowledge.

1.2.4 Way of understanding conception of tactics of cognitive activities

Considering the aspect of methodological knowledge under the viewpoint of activities, the tactics of cognitive activities could be understood as follows:

Mathematics tactics of cognitive activities are the knowledge on the way of surveying, changing subjects (featured or skillful) to solve specific cases in the mathematics cognitive activities.

It is meant that the tactics of cognitive activities is subject to the students' way of implementation, which the products are obtained by their experience and featured by unique or skillful characteristics. The methodological knowledge is the result from the implementation of tactics of cognitive activities. Such result is applied on a group of subjects, becoming the methodological knowledge. The tactics of cognitive activities is developed and carried out on a group of subjects, becoming the methodological knowledge. Such knowledge is used for solving a specific case during the implementation of mathematics cognitive activities.

The tactics of cognitive activities are the knowledge on the way of thinking, helping the students apprehend knowledge, understand the meaning of knowledge and apply the knowledge to achieve the high performance. Such knowledge arises out when the students face with difficulties and obstacles, helping the students solve such difficulties and obstacles during the implementation of mathematics cognitive activities.

1.2.5 Some specific tactics of cognitive activities

1.2.5.1 Tactics of cognitive activities under knowledge on method of implementing common intelligence activity

a) Tactics of dividing compound objects

The tactics of dividing compound objects is the way of surveying characteristics, relationship of objects in order to classify cleverly a complicated problem into simple ones which could be solved.

Example 1.3. Apply the tactics of dividing compound objects to calculate limit in the form of $\infty.0$: $I = \lim_{x \rightarrow +\infty} x^2 \left(\sqrt{\frac{x+2}{x}} - \sqrt[3]{\frac{x+3}{x}} \right)$

b) Tactics of combination

The tactics of combination is the way of surveying characteristics, relationship of objects in order to combine separate objects into new object which is favorable for problem solving.

Example 1.4. Apply the tactics of combination to solve set of equations:

$$\begin{cases} x^3 - 3x^2 - 9x + 22 = y^3 + 3y^2 - 9y & (1) \\ x^2 + y^2 - x + y = \frac{1}{2} & (2) \end{cases}$$

1.2.4.2 Tactics of cognitive activities under knowledge of implementing logic linguistic activities

a) Tactics of conversion

The tactics of conversion is the way of surveying characteristics, relationship of objects in order to convert in the opposite direction to solve a more favorable case.

Example 1.5. Apply the tactics of conversion to establish the method of finding out limits of function by using definition of derivative.

b) Tactics of changing mathematics problem into other form

The tactics of changing mathematics problem is the way of surveying characteristics, relationship of objects in order to change skillfully an object from a language into other language to solve a more favorable specific case.

Example 1.6. Apply the tactics of changing mathematics problem to solve: “Give three real numbers x, y, z satisfying $x + y + z = 3$. Prove that: $\sqrt{x^2 - x + 1} + \sqrt{y^2 - y + 1} + \sqrt{z^2 - z + 1} \geq 3$ ”.

1.2.5.3 Tactics of cognitive activities under knowledge on implementing common intelligence activity

a) Tactics of using intermediate factors

The tactics of using intermediate factors is the way of surveying characteristics, relationship of objects in order to select skillfully an object as intermediate to solve a more favorable case.

Example 1.7. Apply the tactics of intermediate factors to calculate

$$I = \lim_{x \rightarrow \infty} x^2 \left(\sqrt{\frac{x+2}{x}} - \sqrt[3]{\frac{x+3}{x}} \right).$$

b) Tactics of forming specific case

The tactics of forming specific case is the way of surveying characteristics, relationship of objects in order to form a typical specific case, thereby solves more general issue.

Example 1.8. Apply the tactics of forming specific case to calculate the following limit:

$$\lim_{x \rightarrow 0} \frac{1 - \cos x \cdot \cos 2x \cdot \dots \cdot \cos nx}{x^2}, \quad n \in \mathbb{N}^*$$

c) Tactics of using visual image

The tactics of using visual image is the way of surveying characteristics, relationship of objects in order to present the objects by symbols or images so that the characteristics and their relationships become visual and favorable for seeking solutions.

Example 1.8. Apply the tactics of symbolizing images into concept “sequence limited by 0”.

d) Tactics of using variance of function

The tactics of using variance of function is the way of surveying characteristics, relationship of objects in order to change complicated information and select information in the correlation with function and considering the variance of function to solve requirements more favorably.

Example 1.9. Apply the tactics of using variance of function “Find m so that the following equation has root $x\sqrt{x} + \sqrt{x+12} = m(\sqrt{5-x} + \sqrt{4-x})$ (1)”.

e) Tactics of using graph of function

The tactics of using graph function is the way of surveying characteristics, relationship of objects in order to present information by language of function graph to solve requirements through images of function graph.

Example 1.10. Apply the tactics of using graph “Find $m \in (0; \frac{5}{6})$ so that plan figure limited by function graph $y = \frac{1}{3}x^3 + mx^2 - 2x - 2m - \frac{1}{3}$ and lines $x = 0$, $x = 2$, $y = 0$ has area of 4”.

f) Tactics of using continuity of function

The tactics of using continuity of function is the way of surveying characteristics, relationship of objects in order to select function and use the meaning of continuity of function to solve requirements.

Example 1.11. Apply the continuity of function to solve in-equation:

$$\sqrt{x^2 - 4x + 1} \geq 3\sqrt{x - x - 1} \quad (1).$$

g) Tactics of using monotonousness of function

The tactics of using monotonousness is the way of surveying characteristics, relationship of objects in order to change objects and establish its relationship with monotonous function to solve requirements.

Example 1.12. Apply the monotonousness of function to solve “Give $a \geq b > 0$. Prove $\left(2^a + \frac{1}{2^a}\right)^b \leq \left(2^b + \frac{1}{2^b}\right)^a$ ”.

h) Tactics of mixing variables

The tactics of mixing variables is the way of surveying characteristics, relationship of objects in order to reduce the quantity of variables to help the problem solving more favorably.

Example 1.14. Apply the tactics of mixing variables to solve: “ x, y, z are three numbers under section $[1;4]$ and $x \geq y$, $x \geq z$. Find the minimum value of expression $P = \frac{x}{2x+3y} + \frac{y}{y+z} + \frac{z}{z+x}$ ”.

1.2.6 Characteristics of tactics of cognitive activities

1.2.6.1 The tactics of cognitive activities support the memorization and apprehension of knowledge

Example 1.15. The solution of limit problems must apply the following limits: $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$, $\lim_{x \rightarrow 0} \frac{e^x - 1}{x} = 1$; $\lim_{x \rightarrow 0} \frac{\ln(1+x)}{x} = 1$, sometimes, the students forget such limits. The students may apply the tactics of *conversion* to check their memory.

1.2.6.2 Tactics of cognitive activities which helps shorten the process of problem solving

The tactics of cognitive activities has advantage of shortening the process of thinking about the reasons and could help the implementation of problem solving activities quickly. The tactics of cognitive activities is the way of surveying and changing the featured objects, thereby it could shorten the process of problem solving.

Example 1.16. The tactics of separating the compound objects to shorten the process of problem solving “Find m so that the graph of function $y = \frac{2x-1}{x+1}$ cuts the line d with the angle coefficient m and passes through $A(-2;2)$ at two different points under two branches of graph”.

1.2.6.3 Tactics of conditional cognitive activities

Example 1.17. Find the way of solving the mathematics problem “ $y \in [0;3]$. Find the minimum value of expression

$$f = \sqrt{10x^2 - 10xy + 5y^2} + \sqrt{10x^2 - 26xy + 17y^2} - \frac{\sqrt{10}}{3}y^3 - 2014 ”$$

The obstacle is that the factors included in the expression may be complicated but it is near the expression with vector:

$$P = \sqrt{10x^2 - 10xy + 5y^2} + \sqrt{10x^2 - 26xy + 17y^2}$$

So it arises out the thinking of changing the problem from algebraic into vector problem in coordinate plane.

1.2.6.4 Tactics of cognitive activities connected with each other

The tactics of cognitive activities are dependent on each other. When facing with any problem or solving any duty, the students must cooperate some tactics of cognitive activities with each other.

1.2.7 Presentation of tactics of cognitive activities by students

Level 1. Students recognize the presentation of each tactics of cognitive activities.

Level 2. Students realize the tactics of cognitive activities to solve problems in instructed cases.

Level 3. Students apply the tactics of cognitive activities by themselves to solve problems in specific cases.

1.3 Equipping with tactics of cognitive activities for students in teaching analytics at high schools

1.3.1 Equipping with some ideas for use of tactics of cognitive activities for students

Equipping with ideas of tactics of cognitive activities will help the students recognize deeply the role of tactics of cognitive in apprehension of knowledge, understanding and applying such knowledge.

1.3.2 Equipping with knowledge of tactics of cognitive activities for students

1.3.2.1 Equipping with the methods of applying some tactics of cognitive activities for students

Teachers should equip with structure and methods of applying the tactics of cognitive activities on the basis of specific cases.

1.3.2.2 Equipping with tactics of cognitive activities for students from period to period

Equipping with tactics of cognitive activities for students is a continuous process and spends many periods: diagnosis, creating motivation, understanding nature, application and transfer.

1.3.3 Designing system of some special contents to equip with tactics of cognitive activities for students

Teachers should design exercises systematically in order to facilitate the equipping with tactics of cognitive activities for students. Teaching the tactics of cognitive activities must attach to specific knowledge. The equipping with some tactics of cognitive activities must be planned in details and put into targets of each lesson, as well as teach contents of subjects under curricula and textbooks.

1.4 Content of analytics in mathematics curricula at high schools

1.4.1 Overview of classical analytics

Mathematics analytics is considered as a tool to research functions. The analytics connects closely with geometry and algebra.

1.4.2 Contents and characteristics of analytics in mathematics curricular at current high schools

Limit is the tool to construct derivative which is used for surveying features of function. Anti-derivative is the converted operation of derivative. Analytics is constructed on the basis of derivative and applied in calculation of area and volume.

1.4.3 Opportunities of establishment and development of problem solving competence through teaching analytics

The contents of analytics at high schools could create the opportunities for establishment and development of problem solving competence for the students because the knowledge of analytics may connect with contexts arising out in problem cases and in order to solve such cases, the students are required to survey, discover, collect, process information, recommend and assess measures.

1.4.4 Some tactics of cognitive activities used in analytics at high schools

So that the students could overcome difficulties in analytics, it is required to equip with some suitable tactics of cognitive activities.

1.4.5 Connection between tactics of cognitive activities and problem solving competence in teaching analytics

The tactics of cognitive activities help students carry out effectively the deep survey of measures to recommend new measures, establish new issues, apply such measures to new cases, build methods of mathematics calculation. The tactics of cognitive activities make positive impacts on the problem solving process and increase more the problem solving competence.

1.5 Actual status of teaching analytics at high schools in the direction of improving the problem solving competence through equipping with some tactics of cognitive activities

In the process of teaching contents, teachers have not given the specific purpose of equipping with any tactics for students. The equipping with tactics of cognitive activities has still been on ad hoc basis. Teachers have faced with difficulties in determining the required tactics of cognitive activities, methods of equipping and building system of exercises to equip with tactics of cognitive activities.

The number of students who are able to use tactics of cognitive activities to apprehend and apply knowledge has still been little. Most of students have paid attention to the way of surveying and changing objects in order to understand the concept of analytics, theorem of analytics, features of analytics and effective application of analytics knowledge during mathematics solving.

1.6 Conclusion

Chapter 1 researches theory and practical fundamental of equipping with the tactics of cognitive activities for high school students, with results as follows:

- Clarify issues of problem solving process, problem solving competence and components of problem solving competence.
- Give foundations leading to the understanding of tactics of cognitive activities, example such understanding. Determine some specific tactics of cognitive activities in application of analytics knowledge. Survey characteristics of tactics of cognitive activities, thereby realizing the important role of tactics of cognitive activities during the study and the necessity of equipping with such tactics of cognitive activities for the students together with the apprehension of knowledge. Research some issues of equipping with the tactics of cognitive activities for the students.
- Research contents, objects, targets and characteristics of analytics at current high schools. Determine some tactics of cognitive activities used in analytics at high schools. Relationship between the tactics of cognitive activities and problem solving competence in teaching analytics.
- Survey practice of equipping with the tactics of cognitive activities for students through questionnaire, attending some periods of mathematics, interviewing some teachers.

The theory and practice which have been surveyed and analyzed as above are important foundations for us to offer orientations as well as measures of equipping with the tactics of cognitive activities for the students.

Chapter 2. SOME MEASURES OF TEACHING ANALYTICS AT HIGH SCHOOLS IN THE DIRECTION OF IMPROVING PROBLEM SOLVING

COMPETENCE THROUGH EQUIPPING WITH SOME TACTICS OF COGNITIVE ACTIVITIES

2.1 Orientation of establishing measures of teaching analytics at high schools in the direction of improving problem solving competence through equipping with some tactics of cognitive activities

2.2 Some measures of teaching analytics at high schools in the direction of improving problem solving competence through equipping with some tactics of cognitive activities

2.2.1 Measure 1. Equipping with some tactics of cognitive activities for the students in teaching concepts, theorems, rules and methods

2.2.1.1 Purpose

This measure is aimed at helping the students apprehend effectively concepts, analytics theorems and features through equipping with some tactics of cognitive activities. Establish a foundation of good analytics knowledge to prepare the process of problem solving, and equip with some tactics of cognitive activities for the students to apply in mathematics. Contribute to improve the students' competence of finding, collecting and recording mathematics information.

2.2.1.2 Foundations

According to Tran Kieu: Mathematics knowledge and skills are the foundations of establishing and developing competence through learning mathematics; concurrently he affirmed that the problem solving competence is one of capacities which can be developed for the learners by the mathematics through acquiring concepts, proving mathematics clauses and solving mathematics problems.

Nguyen Ba Kim, affirms that knowledge is not to get for free. The impartation of any knowledge to students is not easy without right methods and ways. So that the students obtain firm foundation of mathematics knowledge, the teachers are required to instruct the students to apprehend such knowledge through the tactics of cognitive activities. With specially designed lessons, the students not only occupy knowledge but also establish measures and methods of occupying such knowledge.

2.2.1.3 Implementation

a) Provide instructions and practice for students to apply the tactics of cognitive activities in learning the concept of Analytics

**) Provide instructions and practice for students to use the tactics of forming specific cases in learning concepts*

According to Nguyen Canh Toan: In the process of solving a topic, theoretical summaries have often not appeared simply; it may require to take into consideration of many special and specific cases, then find out gradually abstract and overview; There are many abstracts which are difficult to find out without suggestions from former specific findings.

Example 2.1. Teachers offer cases with problem, through the tactics of forming specific cases, teaching the cooperation for students in teaching the concepts of increasing function and decreasing function.

Through group discussion in specific cases, the students understand the way of forming specific cases and can form specific cases to understand more deeply the concepts, reinforce the concepts and memorize more sustainably the concepts. The students learn the way of exploiting information from typical cases to understand, memorize concepts and identify cases to apply such concepts.

**) Provide instructions and practice for students to apply the tactics of dividing compound objects*

According to Perkins, the establishment of an efficient concepts requires the systematic supply of explanation of concept, clarify the concept regarding purpose, structure, model and argument.

Example 2.2. Teachers provide instructions for the students to apply the tactics of dividing in classification of information in learning the concepts of maximum value and minimum value of functions.

Read instructions and practice the way of using the tactics of forming specific cases and dividing compound objects in teaching the concepts. The students understand and know the way of applying, concurrently grasp thoroughly ideas of such two tactics to simplify any complicated problem.

**) Provide instructions for students to apply the tactics of using visual images*

According to educator Komensky, in order to have firm knowledge, the visual means are required. The concepts of analytics attach to image of variance and geometry image of graph. The exploitation of information from visual images helps the student find out, identify and discover the connotation and extent of concepts.

Example 2.3. Teaching suggestive oral examination, providing instructions for the students to change images into visual symbols on concept of continuous functions.

Learning the concept attached to visual images helps the students understand nature of concept and memorize such concept more easily. Use of visual image is a tool supporting the students to access and apprehend concepts of analytics. With established images of concepts, it is the basis and materials for the students to connect and use images to solve the given mathematics problems.

**) Provide instructions and practice for students to apply the tactics of conversion*

In mathematics, many knowledges are built naturally through the conversion of thinking process.

Example 2.4. Providing instructions for the students to apply the tactics of conversion to build the concept of primitive function.

b) Provide instructions and practice for students to apply the tactics of cognitive activities in teaching theorems

**) Provide instructions for students to apply the tactics of dividing compound objects*

In teaching theorems, the tactics of division helps emphasize characteristics of theorems and nature which should be noted and memorized in applying the separated theorems so that the students could understand more deeply, memorize longer and avoid mistakes.

**) Provide instructions for students to apply the tactics of using visual images*

In teaching theorems, teachers should instruct the students to apply the tactics of using visual images to present each factor in assumption of theorems through visual images, helping the students realize the meaning of each factor of such assumption; the visual images help the students to give conclusion of theorems. The teachers shall instruct the students through use of visual images of graph to determine logic structure of theorems, understand the role of each factor included in such assumption, thereby the students shall understand more clearly the theorems and apply them to specific cases.

Example 2.5. Apply suggestive oral exams to teach theorem of continuity of function : “If $y = f(x)$ is continuous on section $[a;b]$ and $f(a)f(b) < 0$, it exists at least one point $c \in (a;b)$ so that $f(c) = 0$ ” through equipping with the tactics of using images.

c) Provide instructions for students to apply the tactics of cognitive activities to understand and seek rules and methods

In teaching rules and methods, the teachers should help the students understand, grasp thoroughly rules and methods, and affirm their accuracy. Thereby the students may apply correctly to solve mathematics problems. If the students only know by heart such rules and believe that they are correct, then apply in practice, it will result in mistakes. Therefore, in teaching rules and methods, the teacher must analyze so that the students could understand fully conditions of using such rules. In order to help the students understand such rules, the teachers may instruct the students to apply the tactics to consider, survey and analyze factors and information given in such rules. Thereby the students may understand the logic structure, conditions of application and have basis to believe in the accuracy of such rules and methods.

**) Provide instructions and practice for students to apply the tactics of creating specific cases*

Example 2.6. Organize the teaching in the form of cooperation, instruct the students to find out the rule “find out the minimum value of function on a line segment”.

Creating specific cases helps the students understand the rule and avoid mistakes. With teaching the rules and methods, the teachers may apply the tactics of cognitive activities to provide instructions and practice for the students to connect acquired knowledge to find out rules and methods. The finding of rules and methods will help the students explain the foundation of such rules and methods, understand not only steps of implementation but also nature of such steps.

**) Provide instructions and practice for students to apply the tactics of conversion*

The development history of analytics has shown that, the new concept appears on the basis of giving thinking in opposite direction. The students often do not select u and dv in mathematics problems of calculating integration by part. The tactics of conversion helps the students know the way of thinking suitably to select the most effective way of calculation.

Example 2.8. Instruct the students to apply the method of calculating integration by part to calculate integration. Basis of this method is to apply the

$$\text{formula } \int_a^b u dv = uv \Big|_a^b - \int_a^b v du .$$

Use the *tactics of conversion* to explain the reason of using integration by party and how to use it effectively.

Thus, equipping with the tactics of cognitive activities will benefit the apprehension of knowledge. The relationship between tactics and knowledge may support, reinforce and strengthen mutually. The use of tactics in learning will help the students acquire knowledge better, more sustainably and systematically. So that the students could apprehend well knowledge of mathematics, it is required to equip with the tactics for students. The equipping with tactics is not independent but attached to specific contents of mathematics.

Use the tactics of cognitive activities to analyze definitions, theorems, features, establishment of definitions and construction of theorems. The measure has clarified the important role of the tactics of cognitive activities in helping the students be easy to memorize, understand and apply creatively such concepts, theorems and features. In teaching mathematics, it is not only towards the accumulation of knowledge but also help the students be able to think specially; and by the support of such thinking the knowledge is effective and affects positively the intelligence development. The use of tactics of cognitive activities to occupy knowledge helps not only the students to apprehend the entire knowledge either depth and sustainability but also discover the way of using the tactics of cognitive activities and establishing the tactics of cognitive activities. The necessity of teaching mathematics is to merge the apprehension of knowledge and equipping with some tactics of cognitive activities into the unique close process.

2.2.2 Measure 2. Equipping with some tactics of cognitive activities for students in teaching some applications of analytics knowledge through survey and recognition of problems to find solutions

2.2.2.1 Purpose

This measure is aimed at equipping with some tactics of cognitive activities for students, also reinforce knowledge of analytics and increase the students' capacity of applying knowledge of analytics. Provide instructions and practice for students to apply the tactics of cognitive activities in specific cases when realizing activities of surveying problems, find measures and carry out measures of problem solving.

2.2.2.2 Foundations

The tactics of cognitive activities arise out when difficulties or obstacles appear. The teachers should design any case of applying the knowledge of analytics, containing difficulties and obstacles, instructing the students to practice basing on the suitable selection of tactics of cognitive activities. According to Ton Than [81], the competence is only established and developed in activities; in order to develop the creative competence and thinking, it is required to practice the creative thinking for students, of which the most important characteristics is to form new thinking product. According to Nguyen Thi Lan Phuong [60]: "Mechanism of cognitive development is subject to rule "change of quantity leads to change of quality and vice versa", in which "quantity" is the number of problems apprehended in the form of problem solving, "quality" is the competence of solving problems arising out during the study and actual activities". Such opinions of Ton Than and Nguyen Thi Lan Phuong have shown that: Problem solving competence is only established and developed when the students realize the problem solving activities with full "quantity"; and becoming a person who good at problem solving requires the practice of problem solving activities. Through the organization of surveying problems, finding measures, the teachers shall equip with the tactics of dividing compound objects, combination, forming specific cases, change of mathematics problem, use of visual images, use of intermediate, conversion..., then developing the students' competence of understanding and finding measures.

2.2.2.3 Implementation

a) Provide instructions for the students to survey and realize the problems, find measures and implement the measures of problem solving

In teaching in the direction of improving the problem solving competence, it is required to pay attention to improve the components of problem solving competence through problem solving activities. The teachers instruct the students to apply the tactics of cognitive activities as follows:

- Survey and realize the problems in order to analyze and clarify the important meaning of understanding information and finding out solutions.
- Find out measures of problem solving in order to collect and connect information to determine measures and strategies of solution.

To find out the measures, students must survey and recognize the problems. Such two activities shall be repeated during the problem solving. If neither measure is found out, repeat two activities of surveying and recognizing the problems.

b) Provide instructions for the students to practice the application of tactics of cognitive activities

**) Provide instructions for the students to apply tactics of conversion to find limit of sequence by using recurrence relation*

The way of thinking in opposite direction is a normal one used in difficult case in mathematics.

Example 2.10. Find limit of sequence “Give (u_n) determined by $u_1 = 10$ and $u_{n+1} = \frac{u_n}{5} + 3$ (1) with $n \geq 1$. Find $\lim u_n$ ” by the tactics of conversion.

Through example of equipping the students with the idea that if facing with difficulties when solving directly any problem, it is required to change the direction of thinking, arising out the way of indirect thinking in the opposite direction. Equipping with the way of conversion thinking for students in specific case (change the sequence into known simple form). With the flexible way of thinking in the opposite direction, many mathematics problems may be solved easily and quickly.

**) Instruct students to practice the tactics of dividing compound objects in calculation of limit and integral*

Example 2.11. The teachers instruct the students to apply the tactics of dividing compound objects to calculate the limit $I = \lim_{x \rightarrow 2} \frac{\sqrt{2x^2 + 3x + 2} + x^2 - 5x + 2}{x^2 - 4}$.

**) Provide instructions for the students to practice the application of tactics of using images to find conditions of intersection of function graph*

The use of visual images has not only important position in teaching definitions and theorems but also plays significant role in instructing problem solving. When solving analytics problem, symbol of graph is popular and plays important role in finding the solutions.

Example 2.12. Solve “Find m so that the function graph $y = x^3 - 3mx^2 + 3(m^2 - 1)x - (m^2 - 1)$ crosses abscissa axis at 3 differential points with positive abscissa” through the tactics of using visual images.

Purpose of such question through the process of surveying the problem and finding the measures to equip with the tactics of cognitive activities for students

**) Organize the students’ group discussion to practice and apply the tactics of forming specific cases, and tactics of conversion to find conditions of extreme function*

Regarding mathematics problem of finding conditions so that 3 or 4-degree polynomial function (depending on parameter) has extreme point. The teachers

instruct the students to practice of dividing the function graph into separate cases, and then find the solutions.

Example 2.13. The teachers instruct the students to apply the tactics of forming specific cases to solve: “*Find m so that function $y = x^4 + 4mx^3 + 3(m+1)x^2 + 1$ has minimal or maximum points*”.

**) Instruct the students to practice of applying the tactics of intermediate factor to find points of function graph*

Some mathematics problems require determining points of graph in the analytics at high schools. The selection of suitable intermediate factors plays the important role to find unknown factors. The factors included in the mathematics problem benefit and help the change of such mathematics problem to be easier and find the results more quickly.

Example 2.14. The teachers instruct the students’ group discussion to use the intermediate factor as follows:

Case: Determine two points A, B alternatively to be subject to two branches of function graph $y = \frac{2x+3}{x+1}$ so that the length of AB is shortest.

**) Instruct the students to practice of applying the tactics of using graph in the mathematics problem of tangent line via a point.*

Example 2.15. The teachers instruct the students to apply the tactics of cognitive activities through oral exams to find out the solution.

Case: Find A under vertical axis so that it could draw 3 tangent lines via A to function graph $y = x^4 - x^2 + 1$.

In such case the tactics of graph arise out when the students face with difficulty through the normal solution. In order to realize successful such tactics, the students must survey characteristic of function objects and find out the relationship of tangent lines.

**) Instruct the students to practice of applying the tactics of dividing compound objects to calculate integral*

In mathematics at high schools, the calculation of integral has no algorithm as those of the calculation of derivative. When facing with mathematics problem of calculating integral, the students often apply the integral included in table of anti-derivative by changing or varying variables or applying integral by part. To change unpopular integral into popular integration we could apply the tactics of dividing.

Example 2.16. The teachers instruct the students to apply the tactics of dividing to solve: “*Calculate the integral $I = \int_1^2 \frac{x^2 + 3x + 3}{x^3 + 4x^2 + 3x} dx$* ” and it is solved in the general form.

**) Instruct the students to practice of applying the tactics of using intermediate factors in the case of logarithm function according to the formula of calculating integral by part*

Calculation of the integral by the method of integral by part is an important part of chapter of Anti-derivative, Integral and applications. To do that, the students must apply the formula $\int_a^b u dv = uv \Big|_a^b - \int_a^b v du$, in which $u = f(x)$ and $v = g(x)$. Three factors included in the function u , differential dv and du which are unchanged factors, and v which is the determined factor and may be different each other. In order to solve such type of mathematical problem, it is required to analyze carefully the formula under the integral mark to select u and dv suitably. Regarding the application of integral by part, the students must base on characteristics of fixed factor du and select the intermediate factor v suitably to integrate $\int_a^b v du$ simply and easily.

Example 2.17. Instruct the students to apply the tactics of selecting intermediate factor to calculate the integral of logarithm function.

Case: Calculate $I = \int_1^3 \frac{1 + \ln(x+1)}{x^2} dx$

**) Instruct the students to practice of applying the tactics of using variable direction of function in case of applying the analytics to geometry*

Example 2.18. Instruct the students to apply the tactics of using the variable direction through group discussion.

Case: In plane with coordinate system Oxy , $E(3;4)$, line $d: x + y - 1 = 0$ and circle $(C): x^2 + y^2 + 4x - 2y - 4 = 0$. M on d and out of (C) . From M draw tangent lines MA, MB to circle (C) (A, B are tangential points). (E) is centered circle E and contacts to line AB . Find coordinate of point M so that circle (E) has the largest circumference.

**) Instruct the students to practice of applying the tactics of changing mathematics problem to find the minimum and maximum value of expression with many variables*

To solve fluently analytics problems, establishment of tactics of changing mathematics problems for the students is necessary. When solving a mathematics problem, the students must consider and analyze given objective factors, considering them in different problems, such as related to geometry, trigonometry, algebra... Each opinion in the form of different mathematics problems will lead to different solutions.

Example 2.19. Help the students understand clearly the tactics of changing mathematics problems; organize the students' group discussion in the following cases:

Case: x, y, z are actual numbers satisfying $-1-2\sqrt{2} < x < -1+2\sqrt{2}$, $y > 0$, $z > 0$ and $x + y + z = -1$. Find the minimum value of expression $P = \frac{1}{(x+y)^2} + \frac{1}{(x+z)^2} + \frac{1}{8-(y+z)^2}$.

Through the cases of applying the analytics knowledge, the students experience the use of tactics of cognitive activities to solve difficulties and obstacles in specific cases; then they learn the way of escaping difficulties in specific cases.

Thus, the case of applying analytics knowledge in the direction of improving the problem solving competence could equip with the tactics of cognitive activities for students. Instruct the students to practice the way of using the tactics to find out solutions of specific cases, help the students acquire some tactics of cognitive activities to solve problems and contribute to improve the students' problem solving competence. The effective implementation of such measure requires the teachers to establish and design the case of teaching, detecting typical issue containing difficulties and obstacles which require the use of tactics of cognitive activities to overcome. Without specially designed learning cases, the students shall not understand when the tactics of cognitive activities are used and how to use them.

2.2.3 Measure 3. Select the cases to apply the analytics knowledge for students to practice some tactics of cognitive activities

2.2.2.1 Purpose

This measure practices the students the way of re-consideration, analysis and deep research of problem solving process to assess measures, clarify the way of using some tactics of cognitive activities. Practice the students the flexible application of some tactics of cognitive activities to offer some new measures, establishing new cases and problems, and expanding problems. Practice the students to survey origin of some tactics of cognitive activities for them to apprehend the tactics of cognitive activities more sustainably. Practice the students the capacity of forming some tactics of problem solving to solve the problems during their study of mathematics.

2.2.1.2 Foundations

The assessment of problem solving measures may inspire the students to learn mathematics, affect positively their process of learning mathematics. The assessment of problem solving measures and expansion of problems are the most important phases of problem solving, which are activities of providing opportunities of developing the competence of detection and problem solving and creating of students. Polya affirms that considering the way of finding, surveying and analyzing results again, the students could reinforce their knowledge and develop their ability of solving mathematics problems. According to Polya [58]

most of good results of mathematics problems may lose if the students fail to re-consider, survey or analyze again the way of solving mathematics problems.

Instruct the students to apply tactics to assess measures, find the new way of solutions, develop methods of solutions, create new mathematics problems to help them reinforce and grasp thoroughly their knowledge, establish the method of learning mathematics. Reconsideration, deep survey and analysis of measures have important meaning to the students. When the students re-analyze the given measures to help the students summarize used tactics, reconsider the way of using such tactics, then apprehend the way of applying tactics. On the basis of solved issues, the students may skillfully apply such tactics to shorten solutions.

2.2.1.3 Implementation

a) *Practice the students' capacity of assessing measures to recommend new measures on the basis of applying some tactics of cognitive activities*

Nguyen Canh Toan affirms that many mathematics inventions, including important inventions, are originated from the fact that the inventors have new viewpoint of old and popular thing to the extent it seems there is nothing to find out and exploit on such "separate thing". Many measures are popular if the teachers know the way of instructing the students to research such measures. The research of measures helps the students find out the way of solving difficulties of measures and apply the tactics of cognitive activities to solve problems and recommend new substitute measures.

Example 2.21. Organize the students' group discussion to assess the measures and recommend the substitute measures to calculate limit of function which contain degree-2 radical.

Case: Calculate $I_1 = \lim_{x \rightarrow 2} \frac{\sqrt{x+7}-3}{x-2}$, as follows:

$$I_1 = \lim_{x \rightarrow 2} \frac{x-2}{(x-2)(\sqrt{x+7}+3)} = \lim_{x \rightarrow 2} \frac{1}{\sqrt{x+7}+3} = \frac{1}{6}$$

Newly recommended measure:

$$\text{Measure 1. } I_1 = \lim_{x \rightarrow 2} \frac{\sqrt{x+7}-3}{\sqrt{x+7}^2-3^2} = \lim_{x \rightarrow 2} \frac{\sqrt{x+7}-3}{(\sqrt{x+7}-3)(\sqrt{x+7}+3)} = \lim_{x \rightarrow 2} \frac{1}{\sqrt{x+7}+3} = \frac{1}{6}$$

$$\text{Measure 2. } t = \sqrt{x+7}$$

$$I_1 = \lim_{t \rightarrow 3} \frac{t-3}{t^2-9} = \lim_{t \rightarrow 3} \frac{t-3}{(t-3)(t+3)} = \lim_{t \rightarrow 3} \frac{1}{t+3} = \frac{1}{6}$$

$$\text{Measure 3. } I_1 = \lim_{x \rightarrow 2} \frac{\sqrt{x+7}-3}{x-2} = f'(2) = \frac{1}{2\sqrt{2+7}} = \frac{1}{6}$$

The teachers instruct the students to research the measures of problem solving, survey limits and difficulties in the implementation of measures; then, think of using the tactics of cognitive activities to solve difficulties and recommend new measures.

Example 2.22. Organize the students' group discussion to assess the measures and recommend new measures:

Case: Solve in-equation $(2^{1-x} - 2x + 1)(2^x - 1)(x^2 - 5x + 6) > 0$

After giving a measure, it is required to practice the students to research deeply, analyze measures and assess measures to find the way of shortening steps of argument, improvement and recovery of difficulties and obstacles in solutions and find the shorter new measure.

b) Practice the students' capacity of using the tactics of combining and establishing new issues on the basis of solved issues

The teachers instruct the students to establish new issues on the basis of combining solved simple issues; thereby the students may identify simple issues in complicated cases and know the way of changing the complicated issues into simple one.

Example 2.23. Discuss to establish new mathematics problems on the basis of combining analytics problems of rational fraction function and multinomial function through using the tactics of combination.

c) Practice the students' capacity of applying the tactics of cognitive activities in new cases

After instructing the students the way of applying the tactics of cognitive activities to find out measures in specific cases, the teachers should design new teaching cases to practice the students' capacity of applying some tactics of cognitive activities which have been applied successfully to new cases which have factors close to factors in solved cases.

Example 2.24. Practice the students' capacity of applying the tactics of continuity of function to solve inequation:

Case 1. Solve inequation: $(x^2 - 3x + 2)(16 - 2^x) > 0$

d) Organise the students' research of applying the tactics of cognitive activities to develop the method of solving mathematics problems which apply the analytics knowledge

So that the students could apply some tactics of cognitive activities which establish and develop the methods of solving mathematics problems, the teachers should implement the following steps:

Step 1. Determine the tactics of cognitive activities which should be equipped with for the students.

Step 2. Design cases arising out the tactics of cognitive activities.

Step 3. Establish system of mathematics problems which apply the tactics of cognitive activities.

Step 4. Practice the method of solving mathematics problems for students through the use of tactics of cognitive activities.

Basing on contents of analytics at high schools, the teachers may establish the method of solving mathematics problems which apply the analytics through

organizing and practicing the students' capacity of using the tactics of continuity of functions, tactics of using variable direction, tactics of using monotonousness of function, tactics of using function graph. The teachers may practice the students' capacity of applying the tactics in the following cases:

- Research the tactics of using continuity of function, establishing the method of solving inequation $A(x) > 0$.

- Research the tactics of using the monotonousness of function to develop the method of using the monotonousness of function to solve equation, set of equation and prove inequality.

- Research the tactics of using the variable direction of function to develop and assess through the use of functions in solving equations and inequations.

- Research the tactics of using the variable direction of function to establish the method of using the variable direction of function to solve the mathematics problems of existence of roots of equation.

- Research the tactics of using function graph to establish the method of contacting in proving inequality.

Regarding the mathematics problems which is difficult to apply the acquired knowledge of analytics, the teachers should plan to establish the method of solving mathematics problems for the students, prepare for cases arising out the tactics of cognitive activities, instruct necessary theory for the students when establishing such method, instruct the students to research the documents related to theory and method of solving. Organize to teach oral examinations in combination with group discussion.

After a problem is solved, it is not meant that such issue is closed. The re-analysis of measures of problem solving is very necessary and benefits the students, inspiring the students to think more deeply. The students who could offer good solutions and create new issues are not necessary those who has special intelligence. They also apply the acquired knowledge and similar tactics as by other students but their application is more effective and flexible. If the teachers design the good cases of teaching and instruct the students to practice some tactics of cognitive activities, the students shall apply the tactics of cognitive activities to apprehend knowledge and problem solving and create new issues.

2.3 Conclusion

The measures given in this chapter are aimed at equipping with some tactics of cognitive activities and establish the students' capacity of applying such tactics during their solution of cases which apply analytics knowledge and mathematics learning, leading to positive change during the student' problem solving.

Measure 1 mentions the equipping with tactics of cognitive activities through the students' apprehension of mathematics knowledge in teaching concepts, theorems, rules and methods. Measure 1 helps the students apprehend the knowledge through the application of some tactics of cognitive activities and creating foundations for the students' survey and recognition of issues.

Measure 2 mentions the equipping with the tactics of cognitive activities through the survey and recognition of issues, finding of measures and implementation of measures in the case of applying analytics knowledge and affecting the development of competence of surveying issues and finding measures.

Measure 3 mentions the practice of the students' capacity of researching tactics of cognitive activities to recommend new measures, establish new issue, apply such measures to the new case and develop the method of mathematics problem solving.

The students who are equipped with the tactics of cognitive activities may well connect to form contents of knowledge in study, thereby their acquired knowledge will be sustainable and highly effective. Organize the students' oral exams to solve problems, help the students recognize the difficulties and obstacles, arise out the way of solution through the use of tactics of cognitive activities; thereby both improve the competence of problem solving components and equip with the tactics of cognitive activities for the students.

The given pedagogic measures not only contribute to equip with the way of survey and change for the students to solve the problems given in learnin analytics but also handle with issues given in mathematics. The tactics of cognitive activities are the way as well as necessary means to achieve the purpose of improving the students' competence of problem solving; concurrently increase the performance of learning analytics. The teachers should note the balance between equipping with tactics of cognitive activities and teaching mathematics knowledge.

Chapter 3. PEDAGOGIC TESTING

3.1 Purpose and duties of testing

- *Purpose:* The pedagogic testing is carried out in order to test the scientific assumptions; initially affirm the feasibility and efficiency of pedagogic measures recommended in the thesis through practical teaching. It is detailed as follows:

+ Whether measures recommended in the thesis may be realized during the teaching of analytics at high schools or not?

+ Whether the implementation of measures is equipped with the tactics of cognitive activities for the students, contributing to improving the problem solving competence; concurrently increasing the performance of teaching analytics or not?

- *Duties:*

+ Compose testing documents, instruct the teachers the way of preparation and implementation of periods of analytics teaching in the direction of improving the problem solving competence through equipping with some tactics of cognitive activities for the students.

+ Analyze and process the testing data by the method of statistics of results of equipping with some tactics of cognitive activities for the students.

+ Assess the results of testing under two aspects: feasibility and performance of recommended mesasures.

3.4 Testing results

3.4.1 Qualitative assessment

**) As for students*

- The students are very eager and exciting when they are instructed with the application of tactics of cognitive activities in apprehension of concepts, theorems, features and solution of mathematics problems. The contents become to be easy to understand and acquire. Particularly, the students like the way of using visual images, graphs and table of variables. They have positive attitude of learning. They are more excited in learning and fonding of learning more. They feel happy to learn.

- Capacity of surveying and recognizing the problems; finding measures of problem solving by experiment class better than compared class; deeply researching by experiment class better than compared class. Therefore, it is proven that the students' competence of problem solving at experiment class is more progressive than the compared class.

Therefore, regarding the qualitative aspect, the recommended measures of equipping with tactics of cognitive activities in teaching analytics have initially resulted.

**) As for teachers*

The common assessment of many teachers has shown that it is a very good topic and the teachers are very interest in the establishment of the students' way of flexible and special thinking to find and solve problems. Some teachers who do not teach the experiment are also very interested in the topic. The topic suits to the current trend of education which establishes the students' competences. The teachers who teach experiment are very excited when they apply the measures recommended by trainers. The teachers who teach experiment support and highly appreciate the way of designing the experiment syllabus which presents the simplication of difficult and complicated contents. The knowledge is established naturally and easy to understand. The method of solving difficult mathematics problems is established simply and easily to understand on the basis of exploiting concepts, theorems and exercises. Given examples are good and typically featured.

**) First experiment*

Via qualitative and quantitative analysis of pedagogic testing results in round 1, we can affirm that the inital equipping with the tactics of cognitive activities for the students has positively affected the teachers and students. However, the results have not been as expected because our way of designing syllabus to establish knowledge and method of solving mathematics problems has still been restricted. The system of exercises is designed from easy to difficult level but still been imposed on, not naturally and fine. We have given high requirements against the students' cognition level resulting in the fact that some students have still been faced with difficulties in apprehding some tactics of cognitive activities and applying them to the process of problem solving. Those are issues which we have researched, adjusted and completed in the second experiment.

**) Second experiment*

Learning experiences from the first experiment, in the second experiment, we have known the way of composing syllabus suitable to the students' acknowledgement and actual teaching. The quality of students at experiment class is higher than those of compared class. This has shown that the equipping with tactics of cognitive activities for the student has obtained results.

3.5 Conclusion 3

The duties of pedagogic testing is to verify the scientific assumptions. Particularly whether pedagogic measures which are equipped with some tactics of cognitive activities for the students in teaching analytics in the direction of improving the problem solving competence (mentioned in chapter 2) are feasible and really equipped with the tactics of cognitive activities and contribute to improve the problem solving competence for the student or not. The results obtained from the process of pedagogic testing have shown that:

- The finding and giving measures of equipping with the tactics of cognitive activities for the students in teaching concepts, theorems, rules, methods of solving analytics problems and cases which apply the analytics knowledge are very meaningful and gained the interest of the teachers and students.

- The pedagogic measures may be totally transferred for the teachers to apply to teach analytics at high schools favorably and effectively. With clear purpose of teaching, equipping with some tactics of cognitive activities in teaching concepts, analytics, theorem of analytics, rule and method of solving analytics problems helps the students apprehend knowledge of mathematics more independantly, proactively and actively. The equipping with some tactics of cognitive activities in solving cases which apply the analytics knowledge, help the students solve problems in teaching analytics more successfullt, contribute to improve the problem solving competence and increase the performance of teaching analytics at high shcools.

- Capacity of applying the tactics of cognitive activities when surveying and recognizing a problem, activities of finding measures in case of applying the students' analytics knowledge in compared class better than those of experiment class. The students of experiment class are able to apply the tactics to recommend new measures, recommend new issues, apply to new cases and establish the methods of solving. Quality of exams of students of compared class is higher than those of experiment class. This has shown that the students' problem solving competence has initially been increased. The implementation of such measures requires to equip with tactics of cognitive activities for the students; concurrently contribute to improve the problem solving competence and increase the performance of teaching analytics at high schools.

- Purpose of testing is completed, the performance and feasibility of measures have been affirmed; concurrently the scientific assumptions of the thesis are practically acceptable.

CONCLUSION

The thesis is completed with the desire that through equipping with some tactics of cognitive activities as favorable means for the students' application in solving analytics problems, helping the students' apprehension of analytics knowledge and contributing to increase the performance of analytics. The key results of the thesis include:

1. Systematize issues of problem solving process, teaching problem solving and problem solving competence. Offer opinions of problem solving process, issue, case of inspiring issue, competence, problem solving competence and components of problem solving competence.

2. Systematize researches of tactics of cognitive activities, recommend the way of understanding tactics of cognitive activities. Recommend some groups of tactics of cognitive activities and characteristics of tactics of cognitive activities. Systematize researches of teaching analytics at high schools. Research the actual status of teaching analytics in the direction of equipping with tactics of cognitive activities for students at high schools.

3. Determine orientations of teaching analytics in the direction of improving problem solving competence through equipping with some tactics of cognitive activities. Establish 3 measures of teaching analytics in the direction of improving problem solving competence through equipping with some tactics of cognitive activities, detailed as follows:

Measure 1. Equip with some tactics of cognitive activities during the apprehension of concepts, theorems, rules and methods. This measure is aimed at establishing the students' foundation of good analytics knowledge to prepare the process of problem solving; concurrently thereby the students may acquire the tactics of cognitive activities, change them to be the private assets to apply in mathematics.

Measure 2. Equip with some tactics of cognitive activities for students in teaching some cases of applying the analytics knowledge through survey and recognition of issues, finding measures and implementing such measures of problem solving. This measure is aimed at instructing the students' practice of applying some tactics of cognitive activities through oral exam of problem solving, organizing the students' group discussion of problem solving in order to improve the problem solving competence and increase the performance of applying analytics knowledge.

Measure 3. Practice the students' capacity of researching and using some tactics of cognitive activities in deep research of measures. This measure is aimed at practicing the students' flexible application of some tactics of cognitive activities to find out new measures, establish new issues, apply to new cases and develop the method of mathematics problem solving. Practice the students' capacity of surveying origin of tactics of cognitive activities for the students' apprehension of tactics of cognitive activities more sustainably.

4. Organize the implementation to prove the feasibility and performance of recommended pedagogic measures.

5. In teaching at schools, teaching the tactics of cognitive activities must be carried out concurrently with the establishment of knowledge. Teaching the tactics of cognitive activities is not a separate content but it is equipped naturally together with the process of apprehension of knowledge and skills. Mathematics knowledge must be established by using some tactics of cognitive activities which are taught through contents of teaching at schools.

On the basis of obtained results, it is affirmed that the purpose of research has been achieved; duties of research have been completed and scientific assumptions are acceptable. The thesis has affirmed that equipping with tactics of cognitive activities for the students is extremely necessary, helping increase the performance of teaching analytics in particular and mathematics in general, affecting positively the development of the students' competence of problem solving. This is the trend of research which helps the students establish the way of learning, apprehending knowledge and solving problems in the age when the knowledge continuously increases and it is the reasonable and sound direction, satisfying with the trend of current education which is the establishment and development of the students' competence.