VIBRATION CHARACTERISTIC CHANGES IN DAMAGED STRUCTURE WITH REDUCTION OF CROSS-SECTION

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Abstract

This article presents the study of the change in natural vibration frequency, which is determined with measured acceleration data of the structure when there is a weak reduction in a cross-section. Conducting a simulation with the initial structure and damaged structure has been evaluated the change of natural frequency and mode shape. The results can be used as the basic identification of the location and damage level of the structure. Because of reducing cross-section in different structural positions, the specific vibration form will appear suddenly at different positions. Hence, the level and location of damage can be determined based on the changing of the specific vibration patter.

Keywords: *Natural frequency; mode shape; structural damage; reduction of the cross-section.*

1. Introduction

The performance of the health assessment of the project helps the manager to update the status of the project continuously, detect damage early, and take timely maintenance measures. This leads to significant savings in repair and maintenance costs. Building health assessment is a field attracting the attention of many scientists as well as project managers around the world. Damage identification in the structure from the change of dynamic characteristics has been studied for a long time. When damage occurs in the structure, it changes the vibration characteristics such as natural frequency, vibration pattern, and resistance ratio compared to the undamaged state. In recent years, the damage can be directly diagnosed by various methods, such as through the change of the frequency response function (FRF), change of natural frequency of vibration, change of natural vibration pattern, method of curvature of vibration, and method based on strain energy [1]. The natural frequency of vibration, and the form of the natural vibration can be determined through the measurement data of the dynamic response of the structure (acceleration) [2, 3].

Since the structure's dynamic parameters reflect the structure's actual working state, if there is damage inside the structure, the frequency response of the structure and the dynamic parameters will change. Therefore, after determining the dynamic parameters of the tested structure and comparing them with the dynamic parameters

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calculated for the intact structure, the failure state and the degree of failure of the structure can be determined according to the change of the dynamic parameters of the structure. Depending on the type of structure and location, the degree of damage is different, the law of changing the vibration characteristics is also different. In order to provide accurate methods of determining the damage of the structure, it is necessary to conduct an assessment of changes in dynamic characteristics corresponding to different failure conditions and characteristics.

2. Theoretical basis

2.1. Several methods of damage identification

In the last few years, there have been many methods of determining damage and defects of structures developed based on dynamic parameters with different algorithms and experimental databases, with different advantages and disadvantages. Select more reliable and appropriate Structural Damage Identification Methods (SDIM) is reliable and appropriate, it is necessary to understand the degree of influence of defects and damage on the dynamic properties of structure. Some methods of identifying damage such as:

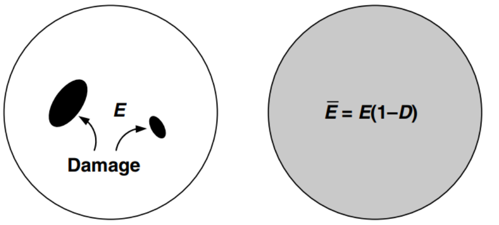
- The method bases on the natural frequency change (Frequency Change - based Damage Detection Method) [4].
- Modal assurance criterion [5].
- Mode Shape Curvature-based Damage Detection Method [6].
- Modal Strain Energy-based Damage Detection Method [7].
- Structural strain modal damage - based on frequency domain decomposition [8].

The location and extent of damage in a structural element is not known in advance. In most practical situations, it is impossible to accurately determine the stiffness at the point of failure. Therefore, as illustrated in Fig. 1, a simple and reasonable approach is to represent for a structural element when damaged, the stiffness is reduced as follows [1]:

$$\overline{EI} = EI(1-D) \tag{1}$$

where E is Young’s modulus for an intact state, I is the moment of inertia, D is the damage magnitude, which is assumed to be uniform over a structure element: $D = 0$ implies the intact state, while $D = 1$ implies complete material rupture due to damage.

Overall damage can increase the nonlinearity of the structure (at the junction between the damaged point and the undamaged point, material nonlinearity will be generated). The initial failures are mainly local at very small points rather than spreading throughout the structure, nonlinearities due to such small overall failures will be weak and limited to small areas around the total failures. Thus, at an early stage of failure development, the resulting nonlinear damage can be ignored.



(a) True damage state (b) Uniform damage representation

Fig. 1. Uniform damage representation of the local damage in a structure element.

2.2. Theoretical basis of vibration characteristic recognition method based on Frequency Domain Decomposition technique

Frequency domain decomposition [3] is a method which decomposes the spectral density matrix at each frequency into singularity values and singularity vectors by the singular value decomposition (SVD). Frequency domain decomposition is an extension of the basic frequency domain technique or commonly known as the Pick Peaking technique, in which natural frequencies are identified by finding peaks in the spectral density matrix.

The relationship between unknown input $x(t)$ and measured response output $y(t)$ can be expressed as follows:

$$[G_{yy}(\omega)] = [H(\omega)]^*[G_{xx}(\omega)][H(\omega)]^T \tag{2}$$

where $[G_{xx}(\omega)]$ is the Power Spectral Density (PSD) matrix of the input, $[G_{yy}(\omega)]$ is the PSD matrix of the responses, $[H(\omega)]^*$ is the complex conjugate matrix of FRF, $[H(\omega)]^T$ is the transpose matrix of FRF.

The FRF can be written in prutial fraction:

$$\left[H(\omega)\right] = \sum_{1}^{N} \frac{[\mathrm{R}_k]}{j\omega - \lambda_k} + \frac{[\mathrm{R}_k]^*}{j\omega - \lambda_k^*} \tag{3}$$

$$\lambda_k = -\sigma_k + j\omega_{dk} \tag{4}$$

where n is the number of modes, λ_k is the pole of the k^{th} mode shape, σ_k is minus the real part of the pole and ω_{dk} is the damped natural frequencies of the k^{th} mode shape. $[\mathrm{R}_k]$ is the residue expressed as follows:

$$[\mathrm{R}_k] = \phi_k . \gamma_k^T \tag{5}$$

where ϕ_k is the mode shape vector, γ_k the modal participation vector.

Suppose the input is white noise, its power spectral density is constant or $[G_{xx}(\omega)] = C$, (C is constant). Formula (2) is rewritten as follows:

$$[G_{yy}(\omega)] = \sum_{1}^{N}\sum_{1}^{N}\left[\frac{[\mathrm{R}_k]}{j\omega-\lambda_k}+\frac{[\mathrm{R}_k]^*}{j\omega-\lambda_k^*}\right].C.\left[\frac{[\mathrm{R}_k]}{j\omega-\lambda_k}+\frac{[\mathrm{R}_k]^*}{j\omega-\lambda_k^*}\right]^T \tag{6}$$

Multiplying the two partial fraction factors and making use of the Heaviside partial fraction theorem, after some mathematical manipulations, the output PSD can be reduced to a pole/residue form as follows:

$$[G_{yy}(\omega)] = \sum_{1}^{N}\frac{[A_k]}{j\omega-\lambda_k}+\frac{[A_k^*]}{j\omega-\lambda_k^*}+\frac{[B_k]}{-j\omega-\lambda_k}+\frac{[B_k^*]}{-j\omega-\lambda_k^*} \tag{7}$$

where $[A_k]$ is the k^{th} residue matrix of the output PSD.

At a certain frequency ω only a limited number of modes will contribute significantly, typically one or two modes. Thus, in the case of a lightly damped structure, the response spectral density can always be written:

$$[G_{yy}(\omega)] = \sum_{k\in Sub(\omega)}\frac{d_k\phi_k\phi_k^{T}}{j\omega-\lambda_k}+\frac{d_k^*\phi_k^*\phi_k^{*T}}{j\omega-\lambda_k^*} \tag{8}$$

where $k \in$ Sub(ω) is the set of modes denoted at a specific frequency, ϕ_k is the mode shape vector, and λ_k is the pole of the k^{th} mode shape.

The frequency domain decomposition technique is based on the singular value decomposition of the Hermitian response spectral density matrix.

$$[G_{yy}(\omega)] = [U][S][U]^H \tag{9}$$

where $[S]$ is a diagonal matrix holding the scalar singular values, $[U]$ is a unitary matrix holding the singular vectors and $[U]^H$ is a Hermitian matrix.

From vibration measurement data of the structure (acceleration), we calculate the spectral density matrix $[G_{yy}(\omega)]$ and decompose the singular value according to formula (9) to determine the natural frequencies, mode shape of the structure.

After determining the dynamic parameters of the tested structure and comparing them with the dynamic parameters calculated for the intact structure, the failure state and degree of damage of the structure can be determined according to the change of structural.

3. Simulation of structure vibration

3.1. Simulation model

Using SAP2000 software to conduct vibration simulation of the structure. The vibration simulation structure is a steel cantilever beam with rectangular section, one end is attached to the other end freely as shown in Fig. 2. The physical parameters of the structure are shown in Tab. 1.

Tab. 1. Test steel beam parameters

No.	Parameter	Symbol	Unit	Value
1	Length	L	mm	710
2	Density weight	ρ	kg/m^3	7850
3	Modulus of elasticity	E	MPa	$2.03 \cdot 10^5$
4	Section width	B	mm	60
5	Section height	H	mm	8

The initial structural diagram without damage is shown as follows:

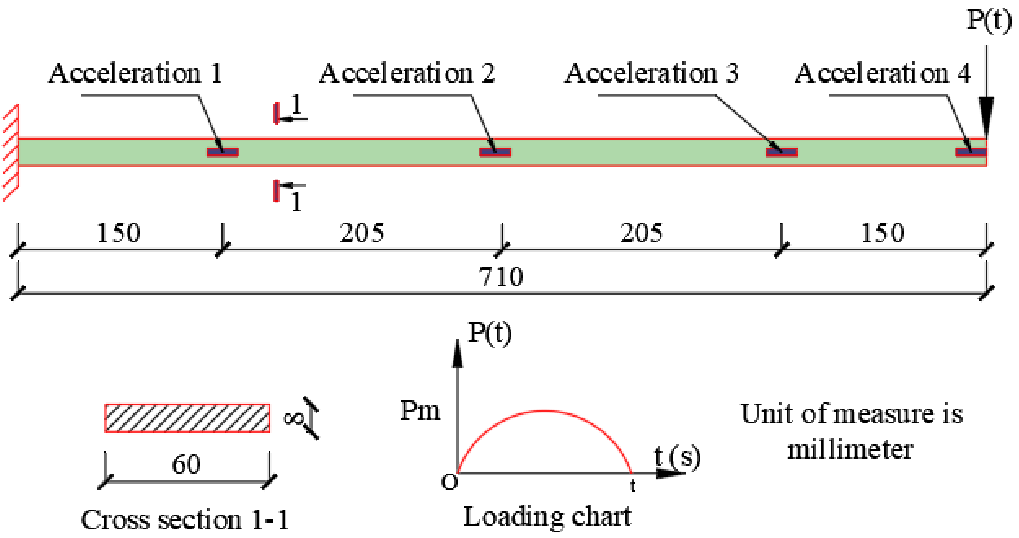


Fig. 2. Intitial structural diagram of beam without damage.

Conduct initial structural vibration simulation when there is no reduced cross section and when the structure has reduced cross-section. Using FDD [2] to identify the natural frequency and natural vibration form from the acceleration value of any two points obtained from SAP2000 software.

Simulation with the case of reduction position at mid-span and reduction at the position at quarter-span from the mount head.

For the two cases, assuming the depth of section reduction is h = 2 mm, the reduction is weak over the entire width of B = 60 mm. Extended weakly reduced length (b) with sizes b = 10 mm, 30 mm, 50 mm, 70 mm, 90 mm.

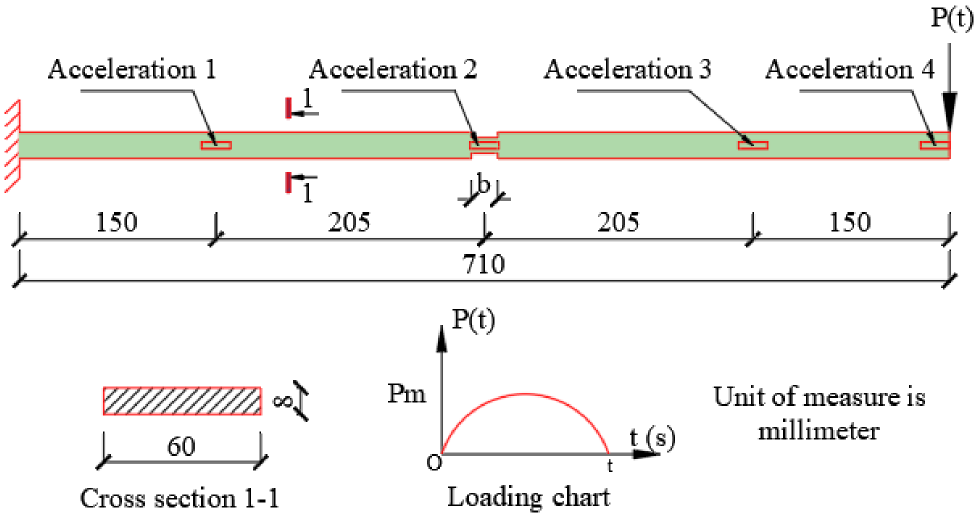


Fig. 3. Structural diagram of beam with reduced cross-section at mid-span.

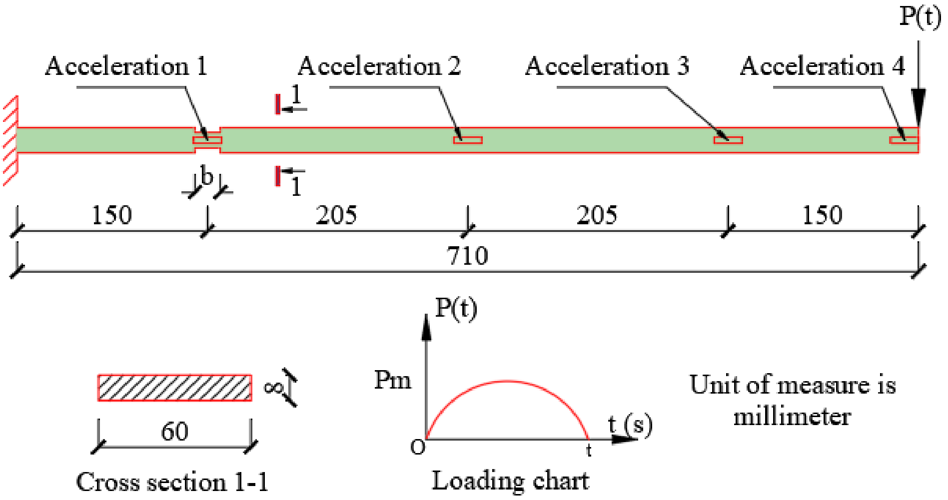
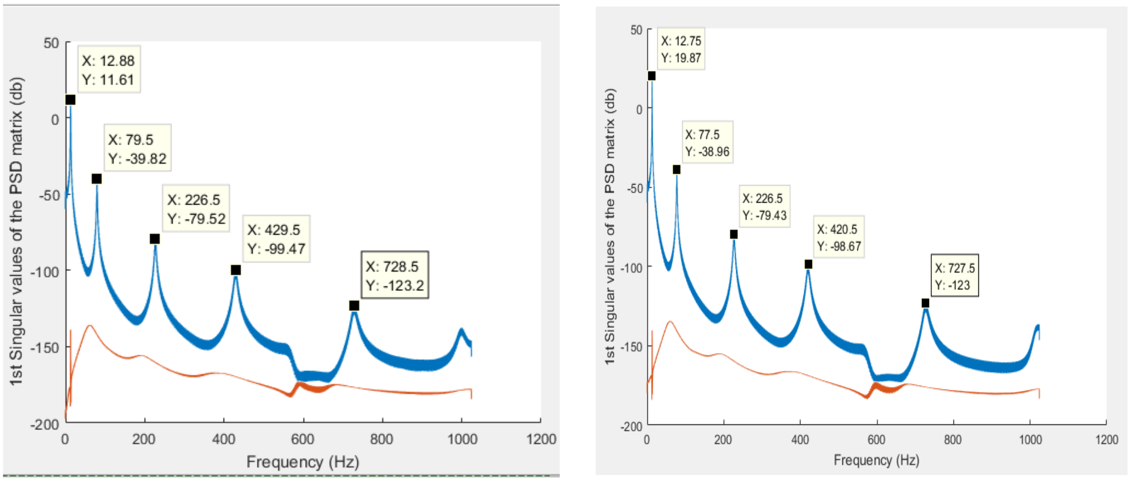


Fig. 4. Structural diagram of beam with reduced cross-section at quarter-span.

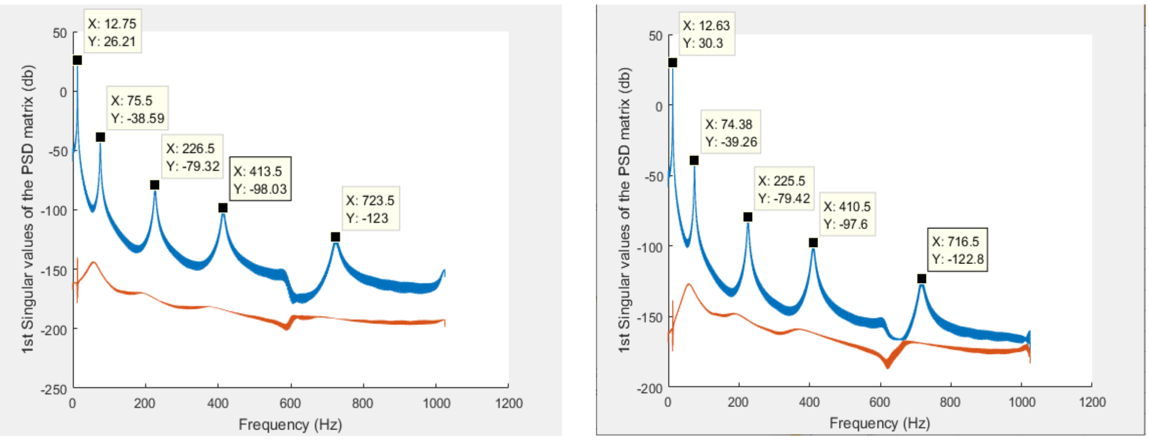
3.2. Analysis results

3.2.1. Results natural frequencies and mode shape of intact and damaged beams at mid-span

- The power spectral density (PSD)



b = 0.01 m, h = 0.002 m *b = 0.03 m, h = 0.002 m*



b = 0.05 m, h = 0.002 m *b = 0.07 m, h = 0.002 m*

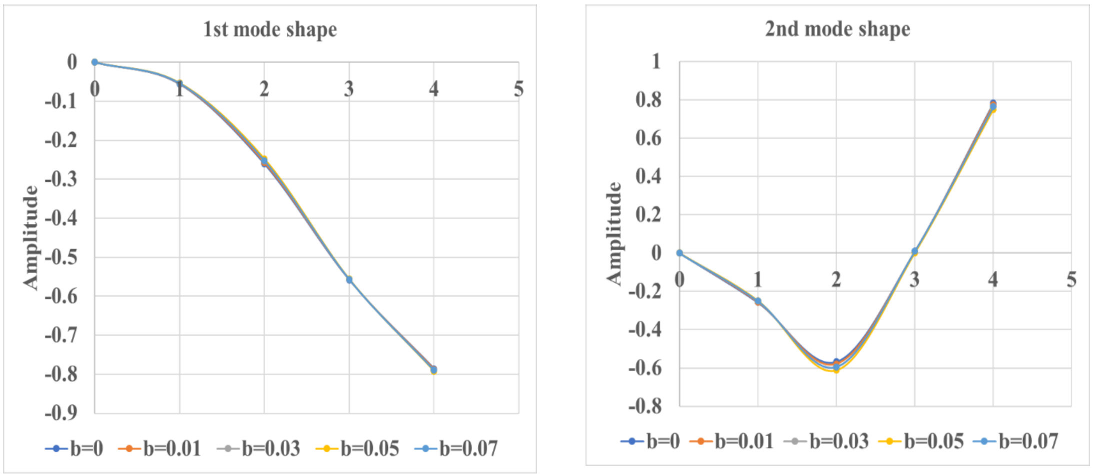
Fig. 5. Power spectrum density of reduced cross-section states.

- Natural frequencies

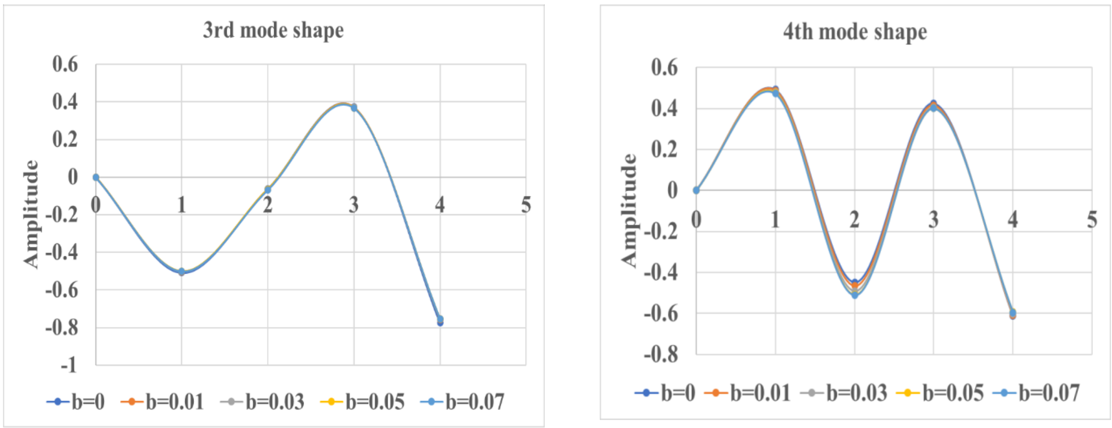
Tab. 2. Results of natural frequencies identification

Mode	**Intact**	**The width of reduced cross-section b (m)**			
		0.01	**0.03**	**0.05**	**0.07**
1	13.25	12.88	12.75	12.75	12.63
2	80.75	79.50	77.50	75.50	74.38
3	226.80	226.50	226.50	226.50	225.50
4	435.50	429.50	420.50	413.50	410.50

- Mode shapes



1^{st} mode shape *2^{nd} mode shape*

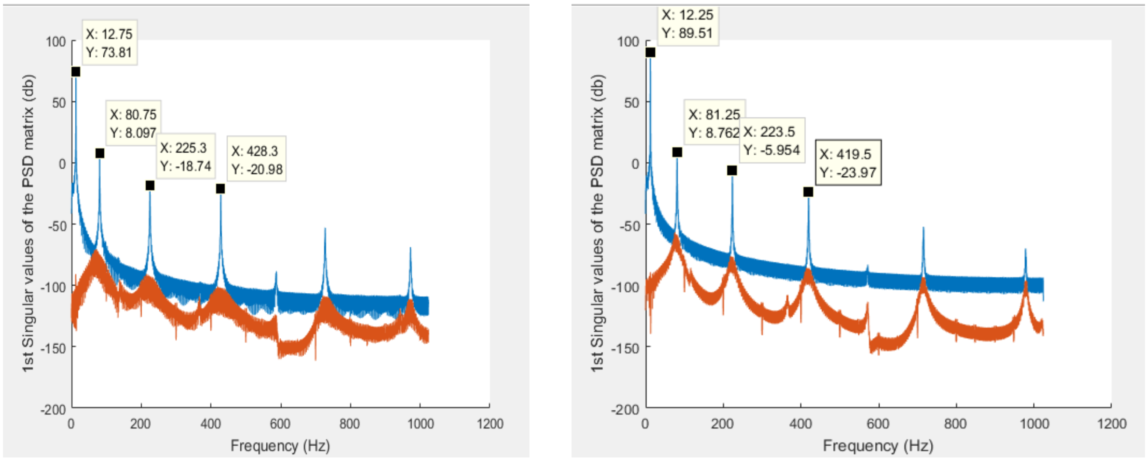


3^{rd} mode shape *4^{th} mode shape*

Fig. 6. Mode shapes reduced cross-section at mid-span.

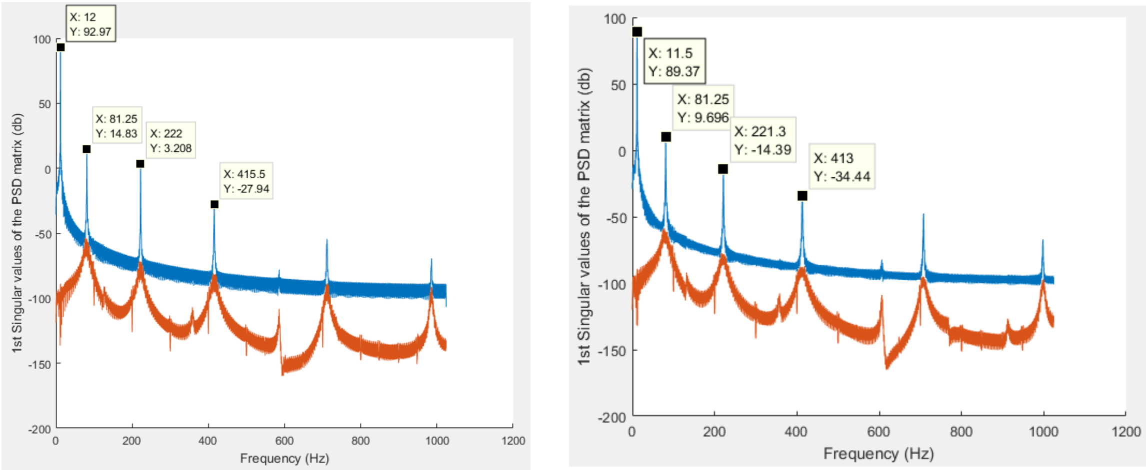
3.2.2. Results natural frequencies and mode shape of intact and damaged beams at quarter-span

- The power spectral density



b = 0.01 m, h = 0.002 m

b = 0.03 m, h = 0.002 m



b = 0.05 m, h = 0.002 m

b = 0.07 m, h = 0.002 m

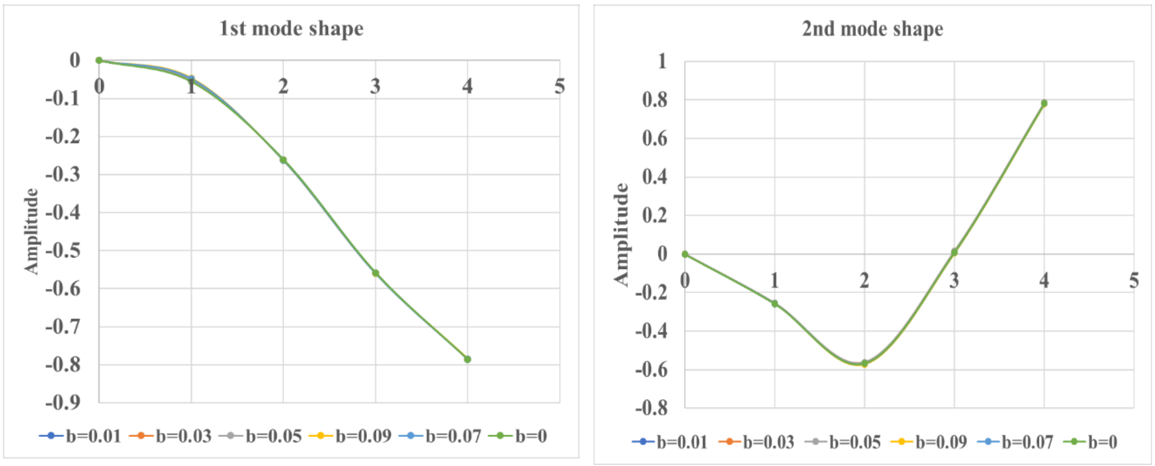
Fig. 7. Power spectrum density of reduced cross-section states.

- Natural frequencies

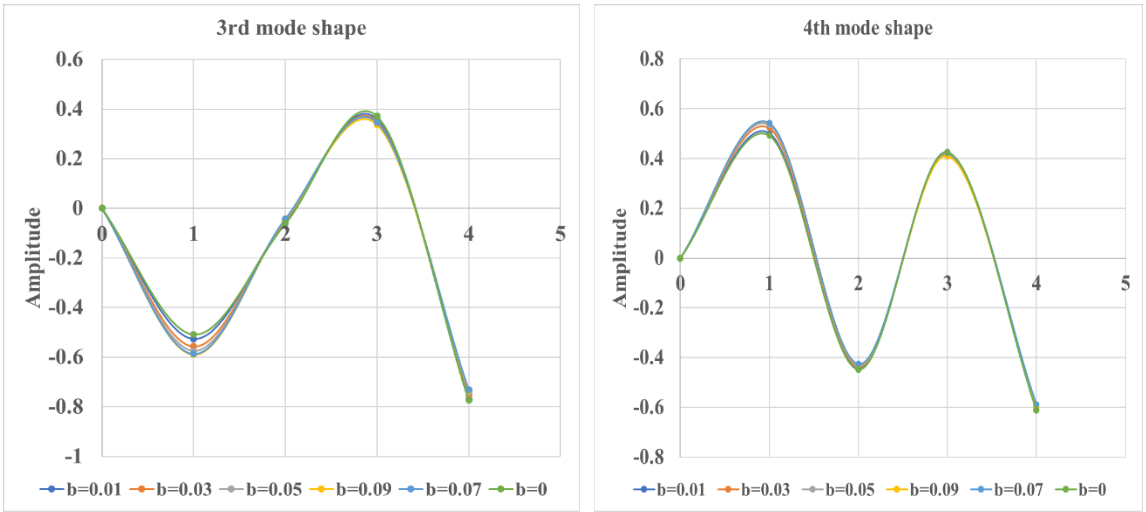
Tab. 3. Results of natural frequencies identification

Mode	**Intact**	**The width of reduced cross-section b (m)**				
		0.01	**0.03**	**0.05**	**0.07**	**0.09**
1	13.25	12.75	12.25	12.00	11.50	11.25
2	80.75	80.75	81.25	81.25	81.25	80.75
3	226.80	225.25	223.50	222.00	221.25	220.30
4	435.50	428.25	419.50	415.50	413.25	412.30

- Mode shapes



1st mode shape *2nd mode shape*



3rd mode shape *4th mode shape*

Fig. 8. Mode shapes reduced cross-section at quarter-span.

Table 2, 3 show that compared with intact structure, the natural frequencies of damaged structure are not significantly different. In general, the natural frequency of damaged structures shows any consistent trend with an increase in the degree of structural damage. The natural frequency variation is obviously not appreciably variable at small damage, and this is consistent with many other studies on the sensitivity of natural frequencies to structural damage.

Figure 6, 8 show the first four-order specific vibration pattern of the test beam with different degrees of failure at quarter-span and mid-span, respectively, according to the FDD method of determining the specific vibration pattern.

- Compared with the intact beam, the test beam is weakly reduced at quarter-span, showing significant changes in the 3rd and 4th natural vibration patterns. This change reflects the existence of structural damage.
- Compared with the intact beam, the test beam is weakly reduced at mid-span, showing significant changes in the 2nd and 4th natural vibration patterns, so this curve also accurately reflects the existence of structural damage.
- Considering all the changes of the vibration pattern at two different fault locations, the mutation of the natural vibration pattern has an important meaning in the identification of the structural failure, this high-frequency natural vibration pattern is more sensitive to structural damage, which is the basis for the construction of damage identification methods.
- When there is a reduction of cross-section in different structural positions, the mutation in the specific vibration form will appear at different positions.

4. Conclusions and recommendations

The results show that the natural frequency of the structure does not change significantly when the damaged structure is small.

Some specific vibration patterns of the structure have a significant change when the structure is weakened and depends on the location of the weakening of the structure.

The location of structural damage can be accurately determined according to the specific vibration mutation, and the intensity of the specific vibration mutation more accurately reflects the different degrees of damage to the structure.

Research results can be used to conduct the development of damage determination methods for structures with complex structures or real buildings under environmental stimuli.

Based on the results of this study, it is possible to rely on the sudden change of the specific vibration pattern to determine the extent and location of damage. Note that, in order to accurately determine the existence and location of structural damage, it is important to choose a reasonable test layout.

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SỰ THAY ĐỔI ĐẶC TRƯNG DAO ĐỘNG VỚI SỰ GIẢM YẾU TIẾT DIỆN KẾT CẤU

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Tóm tắt: *Bài báo trình bày kết quả nghiên cứu về sự thay đổi tần số dao động riêng được xác định thông qua số liệu gia tốc đo được của kết cấu khi có sự giảm yếu tiết diện. Tiến hành mô phỏng với kết cấu ban đầu và kết cấu hư hỏng để đánh giá sự thay đổi tần số dao động riêng, dạng dao động riêng làm cơ sở cho việc nhận dạng vị trí và mức độ hư hỏng của kết cấu. Các kết quả nghiên cứu có thể được sử dụng làm cơ sở để nhận dạng chính xác về vị trí và mức độ hư hỏng của kết cấu. Khi có sự giảm yếu ở các vị trí kết cấu khác nhau thì sự đột biến về dạng dao động riêng sẽ xuất hiện tại các vị trí khác nhau. Do đó, mức độ và vị trí hư hỏng của kết cấu có thể được xác định dựa trên sự thay đổi đột ngột của dạng dao động riêng cụ thể.*

Từ khóa: Tần số dao động riêng; dạng dao động riêng; hư hỏng kết cấu; giảm yếu tiết diện.

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