

Estimating the face-to-face reinforcement ratio of short rectangular reinforced concrete columns based on two uniaxial bendings separately

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Abstract

This paper presents a new technique to design RC columns of the rectangular section, subjected to axial compression and uni- or biaxial bending. Distributed bars around the perimeter are considered as an equivalent thin tube. Using the same steel area for two separate couples of axial compression and uniaxial bending is a trick to solve the equilibrium equations for the optimum steel ratio between faces b and h .

Key words: RC column, bar distribution, uniaxial bending, biaxial bending

1. Introduction

The reinforcement concrete short column (RC column) under biaxial bending can be checked for strength (analysis problem) or designed for reinforcement (design problem). For analysis problem, all parameters of section must be designated, including section shape and dimensions, reinforcement layout, class of concrete and steel. Interaction curve or surface then is established in a process of iterations in which concrete section will be divided into a grid of discrete elements (D. G. Row and T. Paulay in [1] divide the concrete section into a grid of 400 discrete rectangular elements, each with dimensions of $1/20$ of the section width by $1/20$ of the section depth), making it labored work and must usually take advantage of the electronic computer [2]. Some simplified methods based on uniaxial bending have been suggested, such as methods of superposition, equivalent uniaxial eccentricity method [2], reciprocal load method, load contour method [3].

The design problem is more complicated [2] and often limited to reinforcement determination, in which other data, including section parameters, are known. There are two approaches so far: one using a trial and adjustment procedure with computer aid and other using prepared charts. The former required a reinforcement pattern (i.e., the steel bar layout) to be assumed (predefined by software in most cases) and the reinforcement content (i.e., the reinforcement area) successively corrected until the section capacity approached the required value [1][2]. The latter makes use of a large number of charts limited to rectangular sections [2] and has been developed by some researchers. Design charts created by D. G. Row and T. Paulay [1], in which the reinforcement pattern is assumed to be uniformly distributed as a thin tube with $0.25A_{st}$ (total reinforcement area) in each face of the section (for the purpose of computation the reinforcement was still divided into ten discrete units per face [1][2]). Weber charts were made of four, eight, 12, and 16 bar columns (the bars are considered individually rather than as an equivalent thin tube) [1]. Design charts published by Brettle and Warner are for rectangular sections which contain 16 or more bars subject to compression and biaxial bending [1].

The two approaches require a fixed reinforcement pattern: the former uses a program-specific pattern; the latter uses the ratio of A_{sh}/A_{sb} of 1,0 (A_{sh} and A_{sb} are the reinforcement areas of face b and h of the rectangular section, respectively). In fact, reinforcement areas of two adjacent faces should not be equal and are related to internal forces and section dimensions, meaning that there will be a direction that is more force-bearing than the other.

Obviously, there is a need for developing a method to determine the ratio of reinforcement areas of two adjacent faces (termed the face-to-face reinforcement ratio) of a rectangular column that in turn will be the reinforcement pattern for building up the interaction surface.

This article will introduce a way to estimate the questioned ratio, but to its extent will be limited to the case of a column subjected to two combinations of uniaxial bendings.

The thin tube as described by T. Paulay [1] will be used to reduce unknown factors. There are two variables q_b and q_h (substituted for A_{sb} and A_{sh}), that come out in the procedure, in turn requiring two equations corresponding to two selected combinations to solve.

In this article, the ratio of q_h/q_b is termed reinforcement pattern and the value of q_b (or q_h) represents for reinforcement content.

Dividing a concrete section into parts will not create functions, and therefore not appropriate for solving solutions q_b and q_h ; thus the method must start with the creation of functions of the resultant forces, similar to that of circular sections [4].

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Date of receipt: 15/4/2022
Editing date: 6/5/2022
Post approval date: 5/9/2022

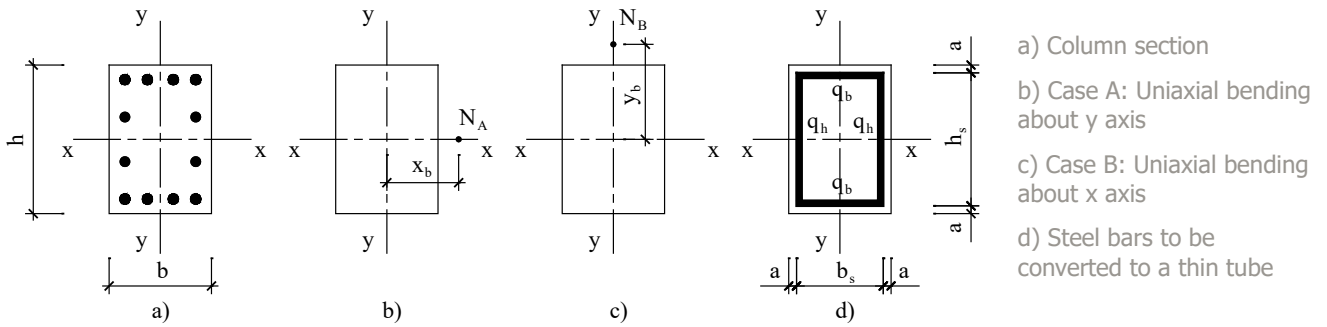


Figure 1: Rectangular RC column section

2. Estimating the face-to-face reinforcement ratio of rectangular RC columns based on two uniaxial bendings separately

Examining a rectangular section with distributed reinforcement as shown in Figure 1a. The section is subjected to two sets of uniaxial forces in different directions, separately, termed case A (NA, MyA) and case B (NB, MxB), as shown in Figures 1b, c.

Translate steel bars on each edge into a thin tube as shown in Figure 1d acquired

$$q_b = \frac{n_b A_{s1}}{b_s}; q_h = \frac{n_h A_{s1}}{h_s} \quad (1)$$

where

nb, nh – number of bar spacings along width (b) and depth (h) of cross section;

As1 – steel bar cross-sectional area;

bs, hs – width and depth of thin tube. bs = b - 2a, hs = h - 2a, where a is the distance from the centroid of the bar to the near edge.

a) Assumptions, sign conventions and scope

The assumptions of reinforcement concrete columns are used in this section, including that plane sections will remain plane after loaded; the tensile strength of the concrete is ignored.

The paper is limited to the following:

Cross section of column is rectangular, reinforcements are provided symmetrically.

The number of reinforcement bars on each edge is large enough to be translated into a thin tube. The number of bars is greater than or equal to five will satisfy this assumption [5].

Axial force is compressive. The strain of outer fiber in the concrete gets its limit of ϵ_{cu} at the ultimate limit state.

Concrete use bi-linear stress-strain relation, steel use tri-linear stress-strain relation.

To begin, formulars of the resultant forces of a typical rectangular section with respect to a neutral axis located at the bottom of the section will be established, as helpers for the next section.

The sign convention of stress/strain is such that compression positive, tension negative.

b) Resultant forces of rectangular section with respect to a neutral axis located at the bottom

It can be inferred from strain distribution $[\epsilon]$ that

$$\frac{\epsilon}{t} = \frac{\epsilon_0}{z} \Leftrightarrow t = \frac{\epsilon z}{\epsilon_0} \quad (2a)$$

$$dt = \frac{z}{\epsilon_0} d\epsilon \quad (2b)$$

Axial force function N and moment function M are

$$N = \int_0^{\epsilon_0} \sigma dA = \frac{bz}{\epsilon_0} \int_0^{\epsilon_0} \sigma d\epsilon = \frac{bz}{\epsilon_0} [P_{(\epsilon_0)} - P_{(0)}] = \frac{bz}{\epsilon_0} [P_{(\epsilon_0)} - 0] = \frac{bz}{\epsilon_0} P_{(\epsilon_0)} \quad (3)$$

$$M = \int_0^{\epsilon_0} t \sigma dA = \frac{bz^2}{\epsilon_0^2} \int_0^{\epsilon_0} \epsilon \sigma d\epsilon = \frac{bz^2}{\epsilon_0^2} [T_{(\epsilon_0)} - T_{(0)}] = \frac{bz^2}{\epsilon_0^2} [T_{(\epsilon_0)} - 0] = \frac{bz^2}{\epsilon_0^2} T_{(\epsilon_0)} \quad (4)$$

where

$$P = \int \sigma d\epsilon; P_{(0)} = 0 \text{ and } T = \int \epsilon \sigma d\epsilon; T_{(0)} = 0 \quad (5)$$

The group of $\sigma(\epsilon), P(\epsilon), T(\epsilon)$ is named a set of functions with respect to strain ϵ .

c) Resultant forces of rectangular RC section

Studying a rectangular RC section with thin tube provided as figure below.

The reinforcement densities of each side are qb and qh, respectively. Section width and depth are b and h. The width and depth of the thin tube are bs and hs. The following expressions are formed

$$\epsilon_{s1} = \frac{z-a}{z} \epsilon_{cu}; \epsilon_{s2} = \frac{z-a-h_s}{z} \epsilon_{cu} \quad (6)$$

c.1 Resultant forces of concrete

Substitute ϵ_{cu} for ϵ_0 in equations (3) and (4), acquired

$$N_b = \frac{bz}{\epsilon_{cu}} P_{(\epsilon_{cu})}; M_b = \frac{bz^2}{\epsilon_{cu}^2} T_{(\epsilon_{cu})} - N_b \left(z - \frac{h}{2} \right) \quad (7)$$

where Mb is the resultant moment of compression zone about centroidal axis (x-x axis).

c.2 Resultant forces of steel

Break thin tube qh into upper part and lower part by neutral axis, still using expressions (3) and (4), acquired

$$N_s = q_b b_s \sigma_{s1} + q_b b_s \sigma_{s2} + 2 \frac{q_h (z-a)}{\epsilon_{s1}} P_{(\epsilon_{s1})} + 2 \frac{q_h (h_s - z + a)}{\epsilon_{s2}} P_{(\epsilon_{s2})} = s_n q_b + u_n q_h \quad (8a)$$

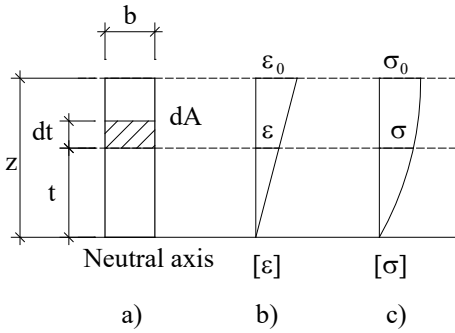


Figure 2: Calculation scheme

- a) Section b) Strain distribution
c) Stress distribution

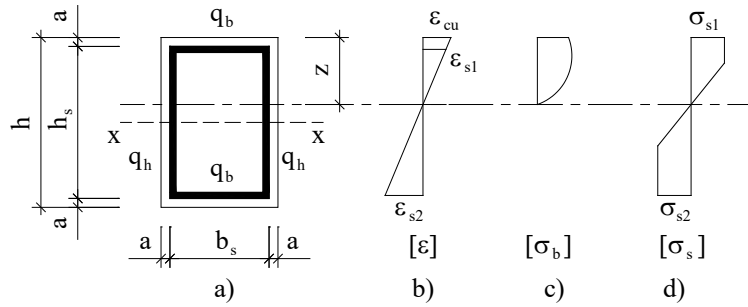
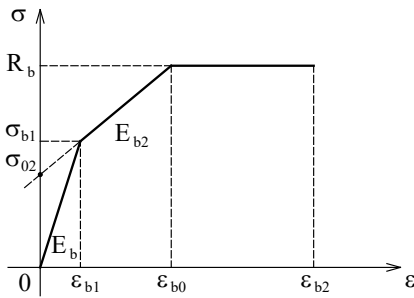
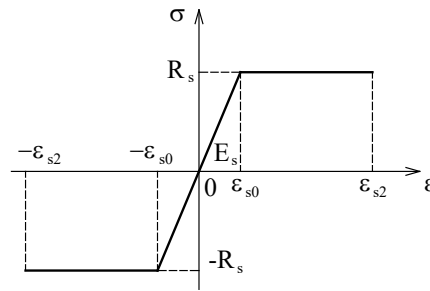


Figure 3: Calculation scheme

- a) Rectangular RC section b) Strain distribution
c) Stress distribution of concrete d) Stress distribution of reinforcement



a) Concrete using tri-linear curve



b) Steel using bi-linear curve

Figure 4: Strain-stress diagram per TCVN 5574-2018

$$M_s = q_b b_s \sigma_{s1} (z - a) + q_b b_s \sigma_{s2} (z - h_s - a) + 2 \frac{q_h (z - a)^2}{\varepsilon_{s1}^2} T_{(\varepsilon_{s1})} - 2 \frac{q_h (h_s - z + a)^2}{\varepsilon_{s2}^2} T_{(\varepsilon_{s2})}$$

$$-N_s \left(z - \frac{h}{2} \right) = s_m q_b + u_m q_h \quad (8b)$$

where N_s and M_s are the resultant force and moment of the thin tube about the centroidal axis, respectively.

$$s_n = b_s (\sigma_{s1} + \sigma_{s2});$$

$$s_m = b_s [\sigma_{s1} (z - a) + \sigma_{s2} (z - h_s - a)] - s_n \left(z - \frac{h}{2} \right) \quad (9a)$$

$$u_n = \frac{2z}{\varepsilon_{cu}} [P_{(\varepsilon_{s1})} - P_{(\varepsilon_{s2})}];$$

$$u_m = \frac{2z^2}{\varepsilon_{cu}^2} [T_{(\varepsilon_{s1})} - T_{(\varepsilon_{s2})}] - u_n \left(z - \frac{h}{2} \right) \quad (9b)$$

c.3 Consequences

Combining (7) and (8) results in

$$N = N_b + s_n q_b + u_n q_h \quad (10a)$$

$$M = M_b + s_m q_b + u_m q_h \quad (10b)$$

There are three variables in the set of equations (10); those are q_b , q_h and compression zone depth z .

d) Function set of concrete and steel as per TCVN 5574-2018 [6]

Relation of stress and strain of concrete and steel is illustrated in Figure 4 below

Based on definitions in (5), the formulars of σ , P , T of concrete and steel are formed and listed in Table 1 and Table 2, respectively.

e) Establishing expression for estimating face-to-face reinforcement ratio

Rewrite (10) using matrix form

$$\begin{bmatrix} N \\ M \end{bmatrix} = \begin{bmatrix} N_b \\ M_b \end{bmatrix} + \begin{bmatrix} s_n & u_n \\ s_m & u_m \end{bmatrix} \begin{bmatrix} q_b \\ q_h \end{bmatrix} \quad (11)$$

Transforming expression (11) produces

$$\begin{bmatrix} q_b \\ q_h \end{bmatrix} = \begin{bmatrix} s_n & u_n \\ s_m & u_m \end{bmatrix}^{-1} \begin{bmatrix} N - N_b \\ M - M_b \end{bmatrix} = \frac{1}{C} \begin{bmatrix} u_m \Delta N - u_n \Delta M \\ -s_m \Delta N + s_n \Delta M \end{bmatrix} \quad (12)$$

where

$$C = s_n u_m - s_m u_n \quad (13a)$$

$$\Delta N = N - N_b \quad (13b)$$

$$\Delta M = M - M_b \quad (13c)$$

For set of forces B (N_B , M_B), bending about x -axis, there are three unknown factors q_b , q_h and compression zone depth y_b

Table 1: Function set of concrete using tri-linear curve

| Functions | $0 \leq \epsilon_b \leq \epsilon_{b1}$ | $\epsilon_{b1} \leq \epsilon_b \leq \epsilon_{b0}$ | $\epsilon_{b0} \leq \epsilon_b \leq \epsilon_{b2}$ |
|-----------|--|--|---|
| σ | $E_c \epsilon_b + \sigma_{b02}$ | $E_{b2} \epsilon_b + \sigma_{b02}$ | R_b |
| P | $\frac{E_c \epsilon_b^2}{2} + \sigma_{b02} \epsilon_b - \frac{\sigma_{b02}^2}{2E_c}$ | $\frac{E_{b2} \epsilon_b^2}{2} + \sigma_{b02} \epsilon_b - \frac{\sigma_{b02} \epsilon_{b1}}{2}$ | $R_b \epsilon_b + \frac{(\epsilon_{b0} - \epsilon_{b1}) \sigma_{b02} - R_b \epsilon_{b0}}{2}$ |
| T | $\frac{E_c \epsilon_b^3}{3}$ | $\frac{E_{b2} \epsilon_b^3}{3} + \frac{\sigma_{b02} \epsilon_b^2}{2} - \frac{\sigma_{b02}^2 \epsilon_{b1}^2}{6}$ | $\frac{R_b \epsilon_b^2}{2} + \frac{(\epsilon_{b0}^2 - \epsilon_{b1}^2) \sigma_{b02} - R_b \epsilon_{b0}^2}{6}$ |
| Notes: | $E_{b2} = \frac{R_b - \sigma_{b1}}{\epsilon_{b0} - \epsilon_{b1}}$ | | $\sigma_{b02} = \frac{\epsilon_{b0} \sigma_{b1} - \epsilon_{b1} R_b}{\epsilon_{b0} - \epsilon_{b1}}$ |

Table 2: Function set of steel using bi-linear curve

| Functions | $-\epsilon_{s2} \leq \epsilon_s \leq -\epsilon_{s0}$ | $\epsilon_{s0} \leq \epsilon_s \leq \epsilon_{s2}$ | $\epsilon_{s0} \leq \epsilon_s \leq \epsilon_{s2}$ |
|-----------|--|--|--|
| σ | $-R_s$ | $E_s \epsilon_s$ | R_s |
| P | $-R_s \epsilon_s - \frac{R_s \epsilon_{s0}}{2}$ | $\frac{E_s \epsilon_s^2}{2}$ | $R_s \epsilon_s - \frac{R_s \epsilon_{s0}}{2}$ |
| T | $\frac{E_s \epsilon_s^3}{3}$ | $\frac{E_s \epsilon_s^3}{3}$ | $\frac{R_s \epsilon_s^2}{2} - \frac{R_s \epsilon_{s0}^2}{6}$ |

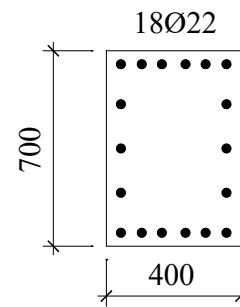


Figure 5: Steel bar layout

$$\begin{bmatrix} q_{bB} \\ q_{hB} \end{bmatrix} = \begin{bmatrix} (u_{mB} \Delta N_B - u_{nB} \Delta M_B) / C_B \\ (-s_{mB} \Delta N_B + s_{nB} \Delta M_B) / C_B \end{bmatrix} \quad (14)$$

For set of forces A (NA, MyA), bending about y-axis, there are three unknown factors qb, qh and compression zone depth xb, noted the swap of qb and qh in cases A and B

$$\begin{bmatrix} q_{bA} \\ q_{hA} \end{bmatrix} = \begin{bmatrix} (-s_{mA} \Delta N_A + s_{nA} \Delta M_A) / C_A \\ (u_{mA} \Delta N_A - u_{nA} \Delta M_A) / C_A \end{bmatrix} \quad (15)$$

Because the values of expression (14) and (15) are the same, then

$$\Delta q_b = q_{bA} - q_{bB} = \frac{-s_{mA} \Delta N_A + s_{nA} \Delta M_A}{C_A} - \frac{u_{mB} \Delta N_B - u_{nB} \Delta M_B}{C_B} = 0 \quad (16a)$$

$$\Delta q_h = q_{hA} - q_{hB} = \frac{u_{mA} \Delta N_A - u_{nA} \Delta M_A}{C_A} - \frac{-s_{mB} \Delta N_B + s_{nB} \Delta M_B}{C_B} = 0 \quad (16b)$$

There are only two variables, xb and yb, in set of equations (16). Solve this for xb and yb, in respect of case A and B, respectively; now that NbA, snA, unA, MbA, smA, umA are taken, thence qb, qh will be determined from expression (15).

The face-to-face reinforcement ratio can now be expressed by thin tube densities or by reinforcement areas:

$$k_q = \frac{q_h}{q_b} \text{ or } k_s = \frac{A_{sh}}{A_{sb}}$$

Set of equations (16) are non-linear systems. Moreover, functions such as Nb, sn, un, Mb, sm, um are piecewise-

defined functions so that to solve exactly for solutions is impossible. In practice, approximate root-finding algorithms can be adopted, such as the trial-and-error method, the Newton-Raphson method...

3. Illustration

A rectangular column section of 400x700mm is subjected to five load combinations as shown in Table 3. Concrete utilizes class B25; the grade of steel is CB400-V. Distance from centroid of steel bar to the near edge is 25mm. Ask for section reinforcement.

Table 3: Design forces

| Combo name | Combo discription | N (kN) | My (kN.m) | Mx (kN.m) |
|------------|--|--------|--------------|--------------|
| O | 1,0D+1,0L | 4187,6 | 42,6 | 25,2 |
| A | 1,0D+0,9L+0,9W _x | 4066,3 | 312,1 | 28,5 |
| B | 1,0D+0,9L+0,9W _y | 3991,5 | 47,3 | 603,9 |
| C | 1,0D+0,5L+1,0E _x +0,3E _y | 3964,8 | 281,9 | 175,4 |
| D | 1,0D+0,5L+0,3E _x +1,0E _y | 3933,1 | 96,8 | 541,6 |

where D, L, W_x, W_y, E_x, E_y denote for dead load, live load, wind load and earthquake load in X and Y directions, respectively.

Solution:

Because the axial forces in combinations are almost equal, then two combinations corresponding to M_{xmax} and M_{ymax} will be selected; i.e., combos A and B in Table 3. The moments M_x of combo A and My of combo B are insignificant (they were produced due to the irregularity of the plan) and can be ignored, meaning that combos A and B can be treated as two uniaxial bendings in estimating process, fetching the

values of (4066,3kN; 312,1kN.m) for combo A and (3991,5kN; 603,9kN.m) for combo B which will be used as the inputs.

$$b_s = b - 2a = 400 - 2 \times 25 = 350\text{mm};$$

$$h_s = h - 2a = 700 - 2 \times 25 = 650\text{mm}$$

a) Material parameters

$$R_b = 14,5\text{MPa}; E_b = 30000\text{MPa}; \varepsilon_{b0} = 0,002;$$

$$\varepsilon_{cu} = \varepsilon_{b2} = 0,0035; \sigma_{b1} = 0,6R_b = 8,7\text{MPa}$$

$$\varepsilon_{b1} = \frac{\sigma_{b1}}{E_b} = 0,00029; E_{b2} = \frac{R_b - \sigma_{b1}}{\varepsilon_{b0} - \varepsilon_{b1}} = 3392\text{MPa};$$

$$\sigma_{b02} = \frac{\varepsilon_{b0}\sigma_{b1} - \varepsilon_{b1}R_b}{\varepsilon_{b0} - \varepsilon_{b1}} = 7,716\text{MPa}$$

$$R_s = 350\text{MPa}; E_s = 210000\text{MPa};$$

$$\varepsilon_{s0} = \frac{R_s}{E_s} = 0,00167$$

b) Determination of x_b, y_b

Let x_b varies from 0 through 400mm; y_b varies from 0 through 700mm; find out the value pair (x_b, y_b) of (334,9mm; 592,1mm) which reconciles with set of equations (16). For these values, the last calculation step is as follows:

For combo B

- Resultant forces of concrete

$$P_{(\varepsilon_{cu})} = R_b \varepsilon_{cu} + \frac{(\varepsilon_{b0} - \varepsilon_{b1})\sigma_{b02} - R_b \varepsilon_{b0}}{2} = 42,847\text{kN} / \text{m}^2$$

$$T_{(\varepsilon_{cu})} = \frac{R_b \varepsilon_{cu}^2}{2} + \frac{(\varepsilon_{b0}^2 - \varepsilon_{b1}^2)\sigma_{b02} - R_b \varepsilon_{b0}^2}{6} = 0,08418\text{kN} / \text{m}^2$$

$$N_b = \frac{b y_b}{\varepsilon_{cu}} P_{(\varepsilon_{cu})} = 2899,4\text{kN};$$

$$M_b = \frac{b y_b^2}{\varepsilon_{cu}^2} T_{(\varepsilon_{cu})} - N_b \left(y_b - \frac{h}{2} \right) = 261,7\text{kN.m}$$

- Value of reinforcement functions

$$\varepsilon_{s1} = \frac{y_b - a}{y_b} \varepsilon_{cu} = 0,003352;$$

$$\varepsilon_{s2} = \frac{y_b - a - h_s}{y_b} \varepsilon_{cu} = -0,00049$$

$$\sigma_{s1} = R_s = 350\text{MPa}; \sigma_{s2} = E_s \varepsilon_{s2} = -102,91\text{MPa}$$

$$P_{(\varepsilon_{s1})} = R_s \varepsilon_{s1} - \frac{R_s \varepsilon_{s0}}{2} = 880,95\text{kN} / \text{m}^2;$$

$$P_{(\varepsilon_{s2})} = \frac{E_s \varepsilon_{s2}^2}{2} = 25,211\text{kN} / \text{m}^2$$

$$T_{(\varepsilon_{s1})} = \frac{R_s \varepsilon_{s1}^2}{2} - \frac{R_s \varepsilon_{s0}^2}{6} = 1,8036\text{kN} / \text{m}^2;$$

$$T_{(\varepsilon_{s2})} = \frac{E_s \varepsilon_{s2}^3}{3} = -0,008235\text{kN} / \text{m}^2$$

$$s_n = b_s (\sigma_{s1} + \sigma_{s2}) = 86482\text{kN} / \text{m};$$

$$u_n = \frac{2y_b}{\varepsilon_{cu}} [P_{(\varepsilon_{s1})} - P_{(\varepsilon_{s2})}] = 289533\text{kN} / \text{m}$$

$$s_m = b_s [\sigma_{s1} (y_b - a) + \sigma_{s2} (y_b - h_s - a)] - s_n \left(y_b - \frac{h}{2} \right) = 51518\text{kN}$$

$$u_m = \frac{2y_b^2}{\varepsilon_{cu}^2} [T_{(\varepsilon_{s1})} - T_{(\varepsilon_{s2})}] - u_n \left(y_b - \frac{h}{2} \right) = 33610\text{kN}$$

$$C = s_n u_m - s_m u_n = -12010 \times 10^6 \text{kN}^2 / \text{m}$$

- Compute q_b, q_h

$$\Delta N = N - N_b = 1092,2\text{kN};$$

$$\Delta M = M - M_b = 342,2\text{kN.m}$$

$$q_b = \frac{u_m \Delta N - u_n \Delta M}{C} = 5,19\text{mm};$$

$$q_h = \frac{-s_m \Delta N + s_n \Delta M}{C} = 2,22\text{mm}$$

Similar to combo B, with combo A

$$q_b = 5,2\text{mm}; q_h = 2,21\text{mm}$$

With the value pair of $x_b = 334,9\text{mm}, y_b = 592,1\text{mm}$, the values of q_b and q_h in both cases A and B are almost equal (slight differences arise due to calculation inaccuracy), which means that this value pair of (x_b, y_b) is the solution of equation set (16).

c) Reinforcement detailing

Steel reinforcements A_{sb} and A_{sh} along edges b and h are

$$q_b = 5,2\text{mm} \Rightarrow A_{sb} = q_b b_s = 1820\text{mm}^2$$

$$q_h = 2,22\text{mm} \Rightarrow A_{sh} = q_h h_s = 1443\text{mm}^2$$

Select bar size of 22mm, cross-sectional area is 380mm²; then the number of spacing between bars along edges b and h are

$$n_b = \frac{A_{sb}}{A_{s1}} = 4,79 \Rightarrow \begin{cases} n_b = 5 \\ n_h = 4 \end{cases}$$

$$n_h = \frac{A_{sh}}{A_{s1}} = 3,8$$
(17)

The face-to-face reinforcement ratio

$$k_s = \frac{A_{sh}}{A_{sb}} = \frac{n_h}{n_b} = 0,8$$

Reinforcement layout is shown in Figure 5, representing the reinforcement pattern (value of 0,8) and reinforcement content (bar size of 22mm). This is just a preliminary arrangement of steel bars.

As declared in [7], "the critical load combinations may be difficult to discern without methodically checking each combination", so the interaction surface must be established, and for each combination, its force set (N, M_x, M_y) shall be checked. If a combination is over strength, reinforcement

Table 4: Checking for strength through combinations

| Combo name | Design forces | | | Ultimate strength | | | Safety factor |
|------------|---------------|-----------------------|-----------------------|---------------------|------------------------|------------------------|---------------|
| | N (kN) | M _y (kN.m) | M _x (kN.m) | N _u (kN) | M _{uy} (kN.m) | M _{ux} (kN.m) | |
| O | 4187,6 | 42,6 | 25,2 | 6049,7 | 61,5 | 36,4 | 1,445 |
| A | 4066,3 | 312,1 | 28,5 | 4130,3 | 317,0 | 28,9 | 1,016 |
| B | 3991,5 | 47,3 | 603,9 | 4012,2 | 47,5 | 607 | 1,005 |
| C | 3964,8 | 281,9 | 175,4 | 4113,4 | 292,5 | 182 | 1,037 |
| D | 3933,1 | 96,8 | 541,6 | 4061,9 | 100 | 559,3 | 1,033 |

Table 5: Total reinforcement area comparison

| Combo name | $k_{q1} = b_s/h_s = 0,583$ | | | $k_{q2} = q_h/q_b = 0,427$ | | |
|------------|--------------------------------------|--------------------------------------|------------------------------------|--------------------------------------|--------------------------------------|------------------------------------|
| | q _b (mm ² /mm) | q _h (mm ² /mm) | A _{st} (mm ²) | q _b (mm ² /mm) | q _h (mm ² /mm) | A _{st} (mm ²) |
| O | 0,9 | 0,48 | 1257 | 1 | 0,43 | 1261 |
| A | 4,62 | 2,49 | 6064 | 5,24 | 2,24 | 6579 |
| B | 4,91 | 2,64 | 6871 | 5,38 | 2,3 | 6752 |
| C | 4,39 | 2,36 | 6145 | 4,97 | 2,12 | 6233 |
| D | 4,58 | 2,46 | 6407 | 5,03 | 2,15 | 6314 |

content will be adjusted (bar size to be increased) while reinforcement pattern remains, and so on.

The results of checking for strength of the section shown in Figure 5 through combinations are shown in Table 4, in which the overall safety factor is the minimum value of 1,005. Although the provided reinforcement area has been chosen to be greater than required (see expression (17)), the margin is still pretty small (1,005 is very close to 1,0) this is because of the consideration of M_y in addition to combo B (for the scope of the article, the calculation process of the values in Tables 4 and 5 is not expressed here).

The reliability of the face-to-face reinforcement ratio k_s (meaning that its value can cause the total reinforcement area to be minimal) depends on the pick of two initial combinations. If the combination pair dominates the design, the result of the process (reinforcement layout in Figure 5) will be the final one. Sometimes these combinations are evident to be found. Especially in the case of buildings of simple shape plan, under wind load are often considered two directions of loading in company with major and minor axes of the plan. If the structure layout is simple and regular, these lateral loads will each produce a uniaxial bending effect to columns (the bending moment in the other direction still exists but is quite insignificant). Since the combinations which consist of lateral loads are commonly dominant in column design, the two combinations of these wind loads with vertical loads such as dead load and live load, still to be uniaxial bendings, will

govern the design result. This is the case within the example.

The final reinforcement area acquired by the face-to-face reinforcement ratio suggested in the article has considered the effect of loadings, and therefore will in theory be optimized than that in the case of a predefined reinforcement pattern. Table 5 displays the comparison between total reinforcement area with respect to face-to-face reinforcement ratio $k_q = q_h/q_b$ (instead of k_s) of the two cases: k_{q1} represents the case in which total reinforcement area to be divided equally to four faces of the column, i.e. A_{sb}=A_{sh}=A_{st}/4, results in $k_{q1} = (A_{sh}/h_s)/(A_{sb}/b_s) = b_s/h_s$; k_{q2} is obtained via the example, using the calculated values of q_h=2,22mm and q_b=5,2mm (it is more reasonable to compare with calculated values than provided ones). The comparison results

show that total reinforcement area acquired from k_{q2} (6752 mm²) is smaller than that of k_{q1} (6871 mm²), meaning that the face-to-face reinforcement ratio of the example is more optimizing than the specific one (the difference between these two values depends on design forces, section aspect ratio... and will not be discussed here).

4. Conclusions

By describing the interaction curve via piecewise-defined functions of the concrete compression zone and the steel thin tube, the proposed method has handled the problem of two variables q_b and q_h, making the design process possible, thence the required face-to-face reinforcement ratio can be obtained as a helper of the next step of finding reinforcement content.

Because the result is just an initial guess, so the assumption with respect to the number of bars on a face mentioned above is not restricted, meaning that it can be less than five. This is the same way manipulated in Paulay's example [1], in which the calculated results were obtained through charts derived from thin tubes but the final bar layout was set up with four bars at corners.

Besides limitations mentioned above, the article has not dealt with some exceptions in calculation (such as coefficient C of expression (13) equals zero, range of x_b, y_b...). These will be exposed in other documents./.

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