Review methods on predicting sediment scour at downstream of hydraulic works

Le Thi Thu Hien[1]

Abstract: The phenomenon of scouring at downstream of sluice or culvert has engrossed the attention of many researchers due to its importance in ensuring the safety of hydraulic structures. Persistent scouring may lead to exposure of the foundations of these structures, thereby causing a threat to their stability. In this study, three methods, namely: physical model, numerical model, artificial inteligent (AI) approach used to predict scour hole geometry are reviewed. Understanding their limitations, strengths and their basic scope of applicability can help researchers select a sufficient tool in predicting scouring problem.

Keyword: Sediment scour, physical model, numerical model, AI approach.

1. Introduction

Culvert, sluice outlets are types of hydraulic structures that control discharge or upstream water level. The phenomenon of scour near hydraulic structures has engrossed the attention of many researchers due to its importance in ensuring the safety of hydraulic structures. Persistent scouring may lead to exposure of the foundations of these structures, thereby causing a threat to their stability (Aamir & Ahmad, 2016). The knowledge of anticipated local scours geometry has been the main concern of engineers or researchers for years because it is a significant criterion for the proper design of sluice outlet foundation (Galán & González, 2020; Abt et al., 1985; Mendoza et al, 1983; Mendoza, 1984). Hence, predicting local scours after water conveyance structures such as spillways, outlet works, etc., has been widely studied to discover adequate protection solutions for the construction. However, the uncertainty of dependent variables to the scour hole such as bed materials, initial conditions of flow, dimensions of hydraulic structures as well as the availability of auxiliary work is always a big challenge in studying this problem. Physical model, mathematical model and AI approaches have been considered three methodologies in investigating local scour after hydraulic work. All methods have pros and con in predicting.

This paper reviews above methodologies to predict scour geometry after sluice and culvert. This should be helpful for researchers to identify and select the suitable method to study this problem. Understand their limitations, strengths and their basic scope of applicability to simulate local scour after hydraulic construction.

2. Methodologies

2.1. Local scour problem at downstream of sluice and culvert

Maximum scour depth or equilibrium depth (d_s) is the most important parameter of scour geometry, which is studied prevalently by several methods. All of them considered that d_s is function of a) initial hydraulic conditions: input discharge (Q), water depth at upstream (Y_u) and downstream (Y_t); b) geometry of sluice or culvert: open height of sluice gate (a); the length of apron (L); the sharp of culvert: circle, box; the length, slope of culvert, the height of

[1]Division of Hydraulics, Thuyloi University
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culvert (d); c) the available of auxiliary devices: wingwall; blockage; d) bed material information: soil density (ρ_s), mean grain size (d_{50}), standard deviation (σ), type of soil: cohesive and non-cohesive; e) gravity acceleration (g), density of water (ρ) (Figure 1). Besides, some dimensionless parameters are often involved in building the equation of d_s, i.e. Froude number of the jet of water after sluice gate (F): $F = V / \sqrt{gh}$ with V is jet velocity; densimetric Froude number (F_d): $F_d = V / \sqrt{\left(\rho_s / \rho - 1 \right) g d_{50}}$; discharge intensity (DI): $DI = Q / g^{1/2} d^{5/2}$.

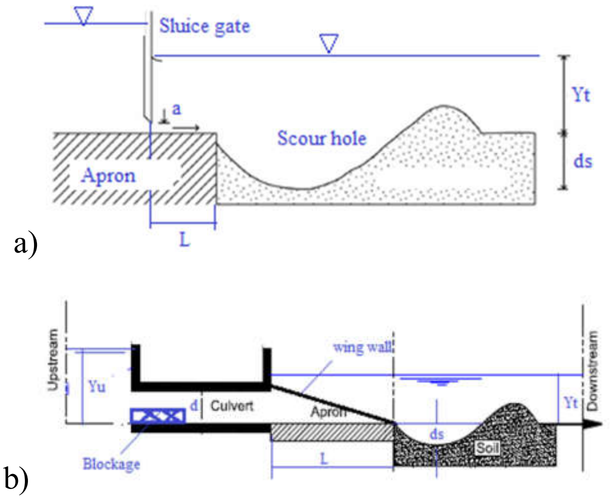

Figure 1. Schematic of sediment geometry after: a) sluice and b) culvert

In general, maximum value d_s after culvert is analyzed and expressed as:

$$\frac{d_s}{d} = \phi\left(\frac{Y_u}{d}, \frac{Y_t}{d}, F_d\right) \text{ or } \frac{d_s}{d} = \phi\left(\frac{Y_u}{d}, \frac{Y_t}{d}, DI\right) \; ; \quad (1)$$

which is base for the experiment campaign. Note that the function ϕ will be different for any combination of culvert shape, culvert outlet configuration and blockage at inlet.

While, this value after sluice gate is also dimensional analyzed:

$$\frac{d_s}{a} = \psi\left(\frac{L}{a}, \frac{Y_t}{a}, F_d\right) \quad (2)$$

Note that the function Ψ will be different for any combination of sluice outlet configuration.

2.2. Physical model

Physical model is considered as traditional method, which are usually used to build empirical equations to calculate maximum scour depth and scour hole geometry (Emami, 2004; Galán & González, 2020; Abt et al., 1985, Abida & Townsend, 1991). However, physical models also exposed several limitations including time-consuming and costly. Especially, it is not flexible or easy to change the dimension or to install auxiliary work as well as the initial conditions, and boundary conditions during experimenting. Besides, the narrow range of physical conditions causes limitations when applying these empirical equations in case studies.

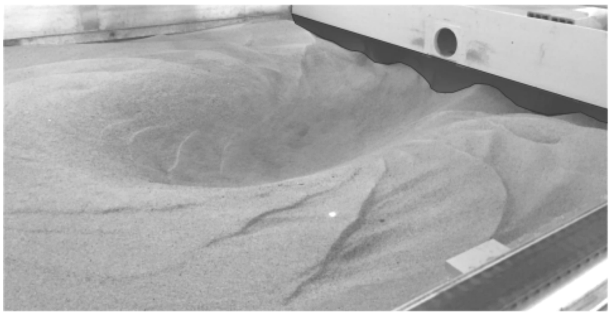

Figure 2. Scour hole after circle culvert (Galán & González, 2020)

The scouring process downstream of an apron is complex in nature owing to the abrupt change of the flow characteristics on the sediment bed with time (Dey & Sarkar, 2006). When the bed shear stress exceeds the critical bed shear stress, the scour initiated at downstream edge of apron. Usually, equilibrium time (t_s) to get steady state of scour hole is also firstly investigated. Then, the dimension of scour geometry in empirical tests are often studied as a function of tail water depth (Y_t); effect of wingwall; effect of culvert sharp; effect of soil properties,

(Figure 2) (Galán & González, 2020). On the other hand, many researchers tried to build the empirical equations in estimating the non dimensionless value (d_s/d) for culvert and (d_s/a) for sluice based on observed data. These equations are efficient tools in predicting scour depth after hydraulic constructions when design these works. However, most of them have limitation range of application due to experimental conditions. Two subsections 2.2.1 and 2.2.2 presented some empirical formula, analytical one taken from published literatures.

2.2.1. Culvert

Table 1. Empirical equations of scour depth after culvert

N°	Investigations	F_d	DI	(Y_t/d)	(d_s/D)
1	Lim (1995)	1.91-2.46	Circle	0.22-7.34	$\frac{d_s}{d}=3.67\left(F_d\right)^{0.57}\left(d_{50}/d\right)^{0.4}\left(\sigma\right)^{-0.4}$
2	Abt et al. (1983)		0.4-3.0	0.45	$\frac{d_s}{d}=2.08\left(DI\right)^{0.37}$
3	Ruff et al. (1984)	7.3-33.7		0; 0.25; 0.45	$\frac{d_s}{D}=2.07\left(DI\right)^{0.45}$
4	Emami & Schleiss. (2010)	7.5-14.5	0.9-1.3	0.15; 1.05	$\frac{d_s}{d}=a\ln\left(F_d\right)+b;\ \begin{cases}a=0.6\left(\frac{Y_t}{d}\right)+1.8\\ b=1.23\left(\frac{Y_t}{d}\right)-2.25\end{cases}$
5	Mendoza et al. (1983)	N/A	Circle		Without wingwall $\frac{d_s}{d}=2.08\left(DI\right)^{0.37}$ With wingwall $\frac{d_s}{d}=2.04\left(DI\right)^{0.36}$
6	Taha et al. (2020)	0.9-2.11	Box	1.25-1.75	$\frac{d_s}{d}=0.56F_d+0.45\left(\frac{Y_t}{d}\right)-1.05$
7	Abida & Townsend (1991)		Box		$\frac{d_s}{d}=\left(\exp\frac{F_d-2}{2.03}-0.373\right)\left(\frac{d_{50}}{d}\right)^{-0.275}$

In seven investigations mentioned in the Table 1, there is only the equation of Mendoza et al., (1983) accounted for the influence of wingwall on scour depth. Therefore, in order to study the effect of dimension of this device or other kinds of auxiliary work on scour geometry in more detail, numerical model should be used (Le et al., 2022).

2.2.2. Sluice gate

Table 2. Equations of maximum scour depth after sluice gate

N°	Investigations	Number of data	d_s/a	d_{50}/a	F	d_s
1	Chatterjee et al. (1994)	28	0.9-1.4	0.02-0.22	1.02-5.46	$\frac{d_s}{a}=0.775F_d$
2	Sarkar & Dey (2005)	38	2.27-8.16	0.02-0.44	2.37-4.87	$\frac{d_s}{a}=0.42F_d^{0.49}\left(\frac{L}{a}\right)^{-0.36}\left(\frac{Y_t}{a}\right)^{1.08}$

N°	Investigations	Number of data	d_s/a	d_{50}/a	F	d_s
3	Dey & Sarkar (2006)	205	2.27-8.16	0.02-0.44	2.37-4.87	$\frac{d_s}{a} = 2.59F_d^{0.94}\left(\frac{L}{a}\right)^{-0.37}\left(\frac{Y_t}{a}\right)^{0.16}\left(\frac{d_{50}}{a}\right)^{0.25}$
4	Hoàng (2012)	N/A				$\xi_m^3 - A\xi_m^2 + B\xi_m + C = 0;\quad \xi_m = d_s / Y_t$; and: $A = 2F\left(0.385V_k^2 - \varepsilon_o\right) + 2$ $B = 2A - 1.54FV_k - 3$ $C = 2F\left(0.77V_k - 0.385V_k^2 - \varepsilon_o + 1.385\right)$ $V_k = \frac{k_o}{F^{1/3}\left(Y_t / d_{50}\right)^{1/6}};$ V_k : *non erosion velocity* (m/s)

The equation of Hoàng (2012) in the Table 2 is the analytical formula, which was generated from boundary layer and jet theory while other are taken from empirical data.

In general, soil types used in almost physical model are non-cohesive. Many tests used only one soil property. Auxiliary devices such as headwall, blockage, apron types like rough or smooth are rarely studies. Besides, due to the lack of quantity of data in many works, the empirical equations extracted from it may be less accurate (Aamir & Ahmad, 2019a). On the other hand, small-scale laboratory experiments have errors caused by scale effect. Because local scour involves complex interactions between sediment, water flow and structures, so it is impossible to ensure all the similarities in a laboratory experiment on scouring (Zhao, 2022).

2.3. Numerical model

Numerical methods have been increasingly used in the study of scour around structures because of their high efficiency and the quickly growing capability of computers for large-scale numerical simulations. Conducting three-dimensional computational fluid dynamics (CFD) simulations can provide a good understanding of vortex structures, which are responsible for scour. Therefore, 3D CFD models solved Navier-Stokes equations by the Volume of Fluid (VOF) method, which is based on the conservation of two mass and momentum, are often used to simulate this problem. A numerous researchers simulated scour hole geometry after culvert or sluice. A number of well-known Computational Fluid Dynamics (CFD) models including OpenFoam, Ansys Fluent, Flow 3D, etc., has been widely utilized in this field (Taha et al., 2020; Elnikhely & Fathy, 2020; Yu et al., 2020; Török et al., 2017). These numerical models based on the coupling of the Volume Fluid Method and Navier Stokes equations have played important roles in simulating sediment scour issue due to the help of state-of-the-art 3D CFD models. The deformation of bed geometry can be demonstrated by the sediment scour module. This model can simulate the sediment transport process, which includes settling, packing, advection, bedload transport, entrainment, and depositions for each species of soil material. Due to the help of a state-of-the-art 3D CFD model, the process of the bed deformation can be performed clearly. So, geometry of scour hole as well as sand mound can be overall predicted. Two mathematical equation systems presented in two subsections 2.4.1 and 2.4.2 are usually solved.

2.3.1. Navier-Stokes equations

In general, bedload and suspended transport are used to describe the movement of sand particles in fluid flow. In the mathematical model, the bed boundary can be considered as a packed one if the local scour occurred at that place. The morphology of the packed boundary is estimated based on the conservation of mass. This process includes bedload transport, absorption, and deposition. The suspended sediment is estimated by sediment concentration and is considered a constraint at each computational cell. For each soil type, this term is estimated by the following continuity equation:

$$V_f \frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u A_x) + \frac{\partial}{\partial y}(\rho v A_y) + \frac{\partial}{\partial z}(\rho w A_z) = 0 \quad (3)$$

where V_f is volume fraction; ρ is fluid density; u, v, and w are velocity components in the x, y, and z directions, respectively; and A_x, A_y, and A_z are the area fractions.

Three momentum equations in the x, y, and z directions are as follows:

$$\begin{aligned}
\frac{\partial u}{\partial t} + \frac{1}{V_F}\left(uA_x \frac{\partial u}{\partial x} + vA_y \frac{\partial u}{\partial y} + wA_x \frac{\partial u}{\partial z} \right) &= -\frac{1}{\rho}\frac{\partial p}{\partial x} + G_x + f_x \\
\frac{\partial v}{\partial t} + \frac{1}{V_F}\left(uA_x \frac{\partial v}{\partial x} + vA_y \frac{\partial v}{\partial y} + wA_x \frac{\partial v}{\partial z} \right) &= -\frac{1}{\rho}\frac{\partial p}{\partial y} + G_y + f_y \\
\frac{\partial w}{\partial t} + \frac{1}{V_F}\left(uA_x \frac{\partial w}{\partial x} + vA_y \frac{\partial w}{\partial y} + wA_x \frac{\partial w}{\partial z} \right) &= -\frac{1}{\rho}\frac{\partial p}{\partial z} + G_z + f_z
\end{aligned} \quad (4)$$

G_x, G_y, and G_z are the body accelerations, and f_x, f_y, and f_z are the viscous accelerations.

2.3.2. Sediment scour model

The sediment transport process is often described by bedload transport and suspended transport. Bedload transport illustrates the motion of soil particles, such as rolling, hopping, and sliding along the packed bed surface due to the shear stress. Bedload transport means the movement of sand particles along the bed channel, regardless of whether some of them become suspended movement. The empirical formulas estimating bedload transport applied in the 3D numerical model were Meyer-Peter Muller, Nielsen, or Van Rijin (Meyer & Müller, 1948; Van Rijn, 1984).

The critical Shields parameter θ_{cr} is used to define the critical bed shear stress τ_{cr}, at which sediment movement begins for both entrainment and bedload transport, which is applied to the horizontal bed.

$$\theta_{cr} = \frac{\tau_{cr}}{g d_{50} (\rho_s - \sigma)} \quad (5)$$

The Soulsby–Whitehouse equation is used to estimate the critical shear stress as follows:

$$\begin{aligned}
\theta_{cr} &= \frac{0.3}{1 + 1.2 d_*} + 0.055\left(1 - e^{-0.02 d_*}\right) \\
d_* &= d_{50}\left[\frac{g(\rho_s / \rho - 1)}{v} \right]^{1/3}
\end{aligned} \quad (6)$$

where υ s the kinematic viscosity of the fluid.

The suspended sediment concentration is calculated by solving the following equation:

$$\frac{\partial C_s}{\partial t} + \nabla.(C_s.\mathbf{u}_s) = \nabla.\nabla(\mathrm{K}.C_s) \quad (7)$$

where C_s is the suspended sediment mass concentration, which is defined as the sediment mass per volume of fluid–sediment mixture; K is the diffusivity; and $\mathbf{u}_s$ is the sediment velocity.

However, due to the application of several hypotheses of the numerical model in simulating sediment transport such as non cohesive soil or the influence of grid size on numerical result, the numerical approach also exposed some limitations in estimating the dimension of the scour hole. In all studies using CFD model, the sediment particles are assumed to be spherical instead of irregular shapes. Considering random shapes of sediment particles in CFD models is still challenging.

Besides, bedload transport equations used in the 3D CFD model are empirical ones, so numerical result is mainly influenced by the selected bedload equation in simulating. Therefore, the numerical parameters should be calibrated and validated by experimental data.

2.4. AI approaches

On the other hand, recently, researchers have expressed keen interest in favor of using soft-computing techniques to predict the scour depth near various hydraulic structures. Some Artificial Intelligence (AI) approaches have been recently applied to predict the maximum scour depth in hydraulic structures (Najafzadeh, 2016; Najafzadeh & Kargar, 2019).

Some artificial intelligence (AI) approaches such as artificial neural networks (ANNs), adaptive neuro-fuzzy inference system (ANFIS), genetic programming (GP), gene expression programming (GEP), group method of data handling (GMDH), and support vector machine (SVM) have been applied to predict the local scour depth at the outlet of culvert (e.g., Liriano & Day, 2001; Azamathulla & Haque, 2012; Najafzadeh, 2016) or sluice (Aamir & Ahmad, 2019b; Najafzadeh & Kargar, 2019; Galán & González, 2020; Najafzadeh & Lim, 2015; Eghbalzadeh et al., 2018; Karbasi & Azamathulla, 2017). In the case of scour depth prediction at the outlets, it should be noted that a large number of studies conducted by AI approaches were a suitable platform in order to reach the scour depth prediction with permissible level of accuracy rather than empirical equations, (Liriano & Day, 2001; Azamathulla & Haque, 2012; Najafzadeh & Lim, 2015). Among mentioned AI models, GP, GEP, and GMDH approaches have the capability of describing a relationship among input and output variables for different realms of scouring problems.

Liriano and Day, (2001) showed that the ANN can successfully predict the depth of scour after culvert with a greater accuracy than existing empirical formulae and over a wider range of conditions. Aamir and Ahmad, (2019a) proved that, empirical equation of Dey and Sarkar, (2006) predicted scour depth after sluice gate with statistical error analysis RMSE value of 0.1 while ANN gave this value for both training and testing 0.05.

However, AI approaches have not been proposed to predict the location of the maximum scour depth and other scour hole geometries (Najafzadeh, 2016). Besides, the accuracy level of predicted equilibrium scour depth taken from this method highly depends on the quantity and quality of databases, which is usually the experimental data.

3. Conclusion

This paper reviewed three methods to predict sediment scour after sluice and culvert. Physical model is considered as a traditional method and still widely used because of its accuracy. It provided database or evidence to other methods. However, it exposed some limitations such as: narrow range of initial conditions in application, inflexible in changing facility, expensive budget and time-consuming. The empirical equations taken from experiment data are efficient and quick tools in predicting the maximum scour depth.

But this result is less accurate than that obtained by AI approach, which is an up-to-date method in predicting equilibrium scour depth. However, the AI result strongly depends on the quantity and quality of database and this method cannot delineate the performance of scour hole. Besides, 3D CFD method performs well the process of sediment transport in the computational domain. It is quite easy and flexible to change initial conditions, boundary condition or substitute auxiliary work. However, both experimental and numerical studies of scour have been mainly focused on inviscid, loose sand. Therefore, based on sediment scour problems and available data as well as the pros and con and application range of each method, the researchers can decide which is the sufficient tool to solve scouring issue.

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